Factorial Notation

a.) $n!$ or $\underline{|n}$ → Read as n- factorial (or) factorial - n

b.) $n! = n \times (n-1) \times (n-2) \times \cdots\cdots \times 3 \times 2 \times 1$
$= 1 \times 2 \times 3 \times \cdots\cdots \times n$

c.) $\underline{|0} = 1$ (defined)

d.) $n = n \cdot (n-1)!$
$= n \cdot (n-1) \cdot (n-2)!$
$= n \cdot (n-1) \cdot (n-2) \cdot (n-3)!$ so - on

Q find the value of following :—

(a) $4! = 4 \times 3 \times 2 \times 1 = 24$

(b) $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 36 \times 20 = 720$

(c) $\frac{8!}{4!} = \frac{8 \times 7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}} = 1680$

(d) $\frac{10!}{8!} = \frac{10 \times 9 \times \cancel{8!}}{\cancel{8!}} = 90$

(e) $\frac{6!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}} = 120$

(f) $\frac{7!}{4!} = \frac{7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}} = 42 \times 5 = 210$

Q. Find $\frac{12! - 10!}{9!}$

Ans:

$$= \frac{12 \times 11 \times 10! - 10!}{9!}$$

$$= \frac{10!\,(12 \times 11 - 1)}{9!}$$

$$= \frac{10! \times 131}{9!}$$

$$= \frac{10 \times 9! \times 131}{9!} = 1310$$

Q. L.C.M. & H.C.F. of 6!, 7!, 8!

Ans:

$8! = 8 \times 7 \times 6!$

$7! = 7 \times 6!$

$6! = 6!$

H.C.F. = 6!

L.C.M. = 8!

Q. $\frac{1}{7!} + \frac{1}{9!} = \frac{x}{10!}$

find the value of 'x'?

Ans: $\Rightarrow x = 10!\left(\frac{1}{7!} + \frac{1}{9!}\right)$

$\Rightarrow x = \frac{10!}{7!} + \frac{10!}{9!}$

$\Rightarrow x = \frac{10 \times 9 \times 8 \times 7!}{7!} + \frac{10 \times 9!}{9!}$

$= 720 + 10 = 730$

Q $(n+3)! = 56 \cdot (n+1)!$

Ans: $\Rightarrow (n+3)(n+2)\cdot(n+1)! = 56 \cdot (n+1)!$

$\Rightarrow (n+3)\cdot(n+2) = 56$

$\Rightarrow (n+3)\cdot(n+2) = 8 \times 7$

$\therefore n = 5$

(P.T.O.)

Q. Prove that $(n!+1)$ is not divisible by any natural number between 2 and n.

Ans: क्योंकी

$$n! = n(n-1)(n-2)(n-3)\cdots 3\cdot 2\cdot 1$$

यह बता रहा है की n! हर natural no. 2 और n के बीच आने वाले से divisible होगा।

इसलिए, $(n!+1)$ जब किसी भी natural no. से divide होगा तो वह 1 as remainder देगा

इसलिए $(n!+1)$ is not divisible by any natural no. b/w 2 and n.

Q. Prove that:-

$$n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

Ans: $L.H.S. = n(n-1)\cdot(n-2)\cdots(n-r+1)$

$$= \frac{n(n-1)(n-2)\cdots(n-r+1)(n-r)!}{(n-r)!}$$

{N^r और D^r दोनों में $(n-r)!$ का Multiply किए}

$$= \frac{n!}{(n-r)!}$$

Q. Prove that :—

$$\frac{(2n!)}{n!} = 2^n \times [1 \times 3 \times 5 \times \cdots \times (2n-1)]$$

Ans: $L.H.S. = \frac{(2n!)}{n!}$

$$= \frac{(2n) \times (2n-1) \times (2n-2) \times \cdots \times 5 \times 4 \times 3 \times 2 \times 1}{n!}$$

अब देखो odd or even terms को n^r मे seperate कर के लिखे।

$$= \frac{\{1 \times 3 \times 5 \times \cdots \times (2n-1)\} \times \{2 \times 4 \times 6 \times \cdots \times (2n-2) \times 2n\}}{n!}$$

$$= \frac{(1 \times 3 \times 5 \times \cdots \times (2n-1)) \cdot 2^n \cdot (1 \times 2 \times 3 \times \cdots \times (2n-1) \times 2n)}{n!}$$

$$= \frac{\{1 \times 3 \times 5 \times \cdots \times (2n-1)\} \times 2^n \times n!}{n!}$$

$$= 2^n \times \{1 \times 3 \times 5 \times \cdots \times (2n-1)\}$$

Q. Prove that

$$\frac{1}{10!} + \frac{1}{11!} + \frac{1}{12!} = \frac{145}{12!}$$

Ans: L.H.S.

$$= \left\{\frac{1}{10!} + \frac{1}{11 \times 10!}\right\} + \frac{1}{12!}$$

$$= \left\{\frac{11+1}{11 \times 10!}\right\} + \frac{1}{12 \times 11!}$$

$$= \frac{12}{11!} + \frac{1}{12 \times 11!}$$

$$= \frac{144+1}{12!}$$

$$= \frac{145}{12!} = \text{R.H.S.}$$

Q. If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, find the value of 'x'.

Ans:

$\Rightarrow x = 8!\left(\frac{1}{6!} + \frac{1}{7!}\right)$

$\Rightarrow x = \frac{8!}{6!} + \frac{8!}{7!}$

$\Rightarrow x = \frac{8 \times 7 \times \cancel{6!}}{\cancel{6!}} + \frac{8 \times \cancel{7!}}{\cancel{7!}}$

$\Rightarrow x = 56 + 8$

$= 64$

Q. Which of the following are true or false?

(i) $(2+3)! = 2! + 3!$

Ans: L.H.S. $= 5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 20 \times 6 = 120$

R.H.S. $= (2 \times 1) + (3 \times 2 \times 1)$

$= 2 + 6 = 8$

$\therefore$ L.H.S $\neq$ R.H.S → so it is false.

(ii) $(2\times3)! = (2!)\times(3!)$

Ans:

$$L.H.S. = 6!$$
$$= 6\times5\times4\times3\times2\times1$$
$$= 30\times24$$
$$= 720$$

$$R.H.S. = (2\times1)\times(3\times2\times1)$$
$$= 2\times6$$
$$= 12$$

$\therefore L.H.S. \neq R.H.S.$ false

Q. If $(n+1)! = 12\times(n-1)!$, find the value of n.

Ans:

$$(n+1)! = 12\times(n-1)!$$
$$\Rightarrow (n+1)\cdot n\cdot(n-1)! = 12\times(n-1)!$$
$$\Rightarrow n(n+1) = 3\times4$$

$\therefore n = 3$

Q. If $(n+2)! = 2550 \times n!$, find the value of n.

Ans: $(n+2)! = 2550 \times n!$

$\Rightarrow (n+2)(n+1)(\cancel{n!}) = 51 \times 50 \times \cancel{n!}$

$\Rightarrow (n+2)(n+1) = 51 \times 50$

On comparing we get

$\therefore$ $n = 49$

Q. If $(n+3)! = 56 \times (n+1)!$, find the value of n.

Ans: $\Rightarrow (n+3)(n+2)\cancel{(n+1)!} = 56 \times \cancel{(n+1)!}$

$\Rightarrow (n+3)(n+2) = 8 \times 7$

$n+3 = 8$	$n+2 = 7$
$n = 5$	$n = 5$

$\therefore$ $n = 5$

Q. If $\frac{n!}{(2!) \times (n-2)!} : \frac{n!}{(4!) \times (n-4)!} = 2:1$, find the value of n.

Ans:

$$\Rightarrow \frac{\frac{n!}{(2!) \times (n-2)!}}{\frac{n!}{(4!) \times (n-4)!}} = \frac{2}{1}$$

$$\Rightarrow \frac{n! \times 4! \times (n-4)!}{n! \times 2! \times (n-2)!} = \frac{2}{1}$$

$$\Rightarrow \frac{4 \times 3 \times 2! \times (n-4)!}{2! \times (n-2) \times (n-3) \times (n-4)!} = \frac{2}{1}$$

$$\Rightarrow \frac{6}{(n-2) \times (n-3)} = \frac{1}{1}$$

$$\Rightarrow (n-2) \times (n-3) = 6$$

$$\Rightarrow (n-2) \times (n-3) = 3 \times 2$$

$$\therefore n = 5$$

Q. If $\frac{(2n)!}{(3!) \times (2n-3)!} : \frac{(n!)}{(2!) \times (n-2)!} = 44:3$, find the value of n.

Ans:

$$\Rightarrow \frac{\frac{(2n)!}{(3!) \times (2n-3)!}}{\frac{(n)!}{(2!) \times (n-2)!}} = \frac{44}{3}$$

$$\Rightarrow \frac{(2n)! \times (2!) \times (n-2)!}{(n)! \times (3!) \times (2n-3)!} = \frac{44}{3}$$

$$\Rightarrow \frac{(2n)! \times (n-2)!}{n \times (n-1) \times (n-2)! \times (2n-3)!} = 44$$

$$\Rightarrow \frac{(2n) \times (2n-1) \cdot (2n-2) \cdot (2n-3)!}{n(n-1) \cdot (2n-3)!}$$

$$\Rightarrow (2n-1) \times 4 = 44$$

$$\Rightarrow (2n-1) = 11$$

$$\Rightarrow 2n = 12$$

$$n = 6$$

$\therefore n = 6$

Q. evaluate :-

$\frac{n!}{(r!) \times (n-r)!}$, when $n = 15$ & $r = 12$.

Ans: plugging in given values, we get

$\Rightarrow \frac{15!}{12! \times 3!}$

$\Rightarrow \frac{15 \times 14 \times 13 \times 12!}{12! \times 3 \times 2}$

$\Rightarrow 35 \times 13$

$\Rightarrow 455$

Q. Prove that

$(n+2) \times (n!) = (n!) + (n+1)!$

Ans: R.H.S. $= n! + (n+1)!$

$= n! + (n+1)\,n!$

$= n!\,(n+1+1)$

$= n! \times (n+2) =$ L.H.S.

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$\therefore$ L.H.S. = R.H.S.

Q. Prove that :—

(i) $\frac{n!}{r!} = n.(n-1).(n-2) \cdots\cdots (r+1)$.

Ans: R.H.S,

$= n.(n-1).(n-2) \cdots\cdots (r+1)$

Multiplying nr & dr by r!

$= \frac{n.(n-1)(n-2) \cdots\cdots (r+1).r!}{r!}$

$= \frac{n!}{r!} =$ L.H.S. $\therefore$ L.H.S = R.H.S

(ii) $(n-r+1). \frac{n!}{(n-r+1)!} = \frac{n!}{(n-r)!}$

Ans: L.H.S $= (n-r+1) \times \frac{n!}{(n-r+1) \times (n-r)!}$

$= \frac{n!}{(n-r)!} =$ R.H.S.

$\therefore$ L.H.S = R.H.S.

(iii) $\frac{n!}{r!\cdot(n-r)!} + \frac{n!}{(r-1)!\cdot(n-r+1)!} = \frac{(n+1)!}{r!\,(n-r+1)!}$

Ans: L.H.S.

$= \frac{n!}{r\cdot(r-1)!\cdot(n-r)!} + \frac{n!}{(r-1)!\cdot(n-r+1)\cdot(n-r)!}$

$= \frac{n!}{(r-1)!\cdot(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$

$= \frac{n!}{(r-1)!\cdot(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$

$= \frac{(n+1)!}{(n-r+1)!\,(r!)}$

= R.H.S. hence proved

^nP_r notation

(a.) Denotion forms :-

^nP_r (or) $P(n,r)$ read as (n - p - r)

(b.) ^nP_r = multiply 'r' numbers starting with 'n'.

$$= \frac{n!}{(n-r)!} ; 0 \le r \le n$$

(c.) $^nP_n = n!$

* ^nP_r का मतलब है 'n' से start कर के r no.s लिखिए।

जैसे :- $^{10}P_2 = 10 \times 9$.

$^{10}P_3 = 10 \times 9 \times 8$

$^5P_1 = 5$.

Q. Evaluate :-

(i) $^{12}P_4 = \frac{12!}{(12-4)!} = \frac{12!}{8!}$

$= \frac{12 \times 11 \times 10 \times 9 \times 8!}{8!}$

$= 99 \times 12 \times 10$

$= 11880$

(ii) $^{75}P_2 = \frac{75!}{73!} = \frac{75 \times 74 \times 73!}{73!}$

$= 75 \times 74$

$= 5550.$

(iii) $^{8}P_8 = 8!$

$= (8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)$

$= 40320.$

(P.T.O.)

Q. Find the value of n such that

(i) $^{m}P_5 = 42 \times {}^{m}P_3$, $m > 4$

Ans: $\Rightarrow \frac{^{m}P_5}{^{m}P_3} = 42$

$\Rightarrow \frac{m(m-1)(m-2)(m-3)(m-4)}{m(m-1)(m-2)} = 42$

$\Rightarrow (m-3)(m-4) = 42$

$\Rightarrow (m-3)(m-4) = 7 \times 6$

on Comparing we get

$\therefore m = 10$

(ii) $\frac{^{m}P_4}{^{m-1}P_4} = \frac{5}{3}$, $m > 4$

Ans: $\Rightarrow \frac{m(m-1)(m-2)(m-3)}{(m-1)(m-2)(m-3)(m-4)} = \frac{5}{3}$

$\Rightarrow \frac{n}{(n-4)} = \frac{5}{3}$

$\Rightarrow 3n = 5n - 20$

$\Rightarrow \cancel{-}2n = \cancel{-}20$

$\therefore n = 10$

Q. If $5 \times {}^4P_r = 6 \times {}^5P_{r-1}$, find 'r'.

Ans:

$\Rightarrow 5 \cdot \frac{4!}{(4-r)!} = 6 \times \frac{5!}{(5-r+1)!}$

$\Rightarrow \cancel{5 \times} \frac{\cancel{4!}}{(4-r)!} = \frac{6 \times \cancel{5!}}{(6-r)!}$

$\Rightarrow \frac{(6-r)!}{(4-r)!} = 6$

$\Rightarrow \frac{(6-r)(5-r)\cancel{(4-r)!}}{\cancel{(4-r)!}} = 6$

$\Rightarrow (6-r) \cdot (5-r) = 3 \times 2$

On comparing, we get

$6 - r = 3 \Rightarrow \cancel{-}r = \cancel{-}3$

$-r = 3 - 6$

$\therefore \boxed{r = 3}$

Q. If ${}^{m}P_4 = 2 \times {}^{5}P_3$, find 'm'.

Ans: $\Rightarrow {}^{m}P_4 = 2 \times {}^{5}P_3$

$\Rightarrow m(m-1)(m-2)(m-3) = 2 \times 5 \times 4 \times 3$

$\Rightarrow m(m-1)(m-2)(m-3) = 5 \times 4 \times 3 \times 2$

On comparing, we get

$m = 5.$

Q. (i) If ${}^{15}P_r = 2730$, find the value of 'r'.

Ans: $\Rightarrow {}^{15}P_r = 2730$

$\Rightarrow {}^{15}P_r = 15 \times 182$

$\Rightarrow {}^{15}P_r = 15 \times 14 \times 13$

$\Rightarrow {}^{15}P_r = {}^{15}P_3$

On comparing, we get

$\therefore$ $r = 3$

(iii) If ${}^{10}P_r = 5040$, find the value of r.

Ans:

$\Rightarrow {}^{10}P_r = 5040$

$\Rightarrow {}^{10}P_r = 10 \times 9 \times 56$

$\Rightarrow {}^{10}P_r = 10 \times 9 \times 8 \times 7$

$\Rightarrow {}^{10}P_r = {}^{10}P_4$

On comparing, we get

$\therefore$ $r = 4$

Q. If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = (30800 : 1)$, find 'r'.

Ans: $${}^{56}P_{r+6} = \frac{56!}{[56-(r+6)]!} = \frac{56!}{(50-r)!}$$

& $${}^{54}P_{r+3} = \frac{54!}{[54-(r+3)]!} = \frac{54!}{(51-r)!}$$

$$\therefore \frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!}$$

$$= \frac{56 \times 55 \times (54!) \times (51-r)((50-r)!)}{(50-r)! \times 54!}$$

$$= 56 \times 55 \times (51-r)$$

Thus, $$\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{30800}{1}$$

$$\Rightarrow 56 \times 55 \times (51-r) = 30800$$

$\Rightarrow (51 - r) = 10$

$\therefore$ $r = 41$

Q. If ${}^{2m+1}P_{m-1} : {}^{2m-1}P_{m} = 3:5$, find 'm'.

Ans: We have,

$${}^{2m+1}P_{m-1} : {}^{2m-1}P_{m} = 3:5$$

$$\Rightarrow \frac{(2m+1)!}{\{(2m+1)-(m-1)\}!} : \frac{(2m-1)!}{\{(2m-1)-m\}!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2m+1)!}{(m+2)!} : \frac{(2m-1)!}{(m-1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2m+1)!}{(m+2)!} \times \frac{(m-1)!}{(2m-1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2m+1) \times 2m \times (2m-1)! \times (m-1)!}{(m+2)(m+1)m(m-1)! \times (2m-1)!} = \frac{3}{5}$$

$$\Rightarrow \frac{(2n+1)\times 2}{(n+2)\times(n+1)} = \frac{3}{5}$$

$\Rightarrow 10(2n+1) = 3(n+2)\cdot(n+1)$

$\Rightarrow 20n+10 = 3(n^2+n+2n+2)$

$\Rightarrow 20n+10 = 3n^2+9n+6$

$\Rightarrow 3n^2 - 11n - 4 = 0$

$\Rightarrow (n-4)\cdot(3n+1) = 0$

$\Rightarrow n=4$ [$\because n \neq -\frac{1}{3}$ as n cannot be negative]

Q. If ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11:52$, find 'r'.

Ans: we have,

$${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11:52$$

$$\Rightarrow \frac{22!}{\{22-(r+1)\}!} : \frac{20!}{\{20-(r+2)\}!} = 11:52$$

$$\Rightarrow \frac{22!}{(21-r)!} : \frac{20!}{(18-r)!} = 11 : 52$$

$$\Rightarrow \frac{22!}{(21-r)!} \times \frac{(18-r)!}{20!} = \frac{11}{52}$$

$$\Rightarrow \frac{22 \times 21 \times \cancel{20!} \times \cancel{(18-r)!}}{(21-r)(20-r)(19-r)\cancel{(18-r)!} \times \cancel{20!}} = \frac{11}{52}$$

$$\Rightarrow (19-r)\cdot(20-r)(21-r) = \frac{52 \times \overset{2}{\cancel{22}} \times 21}{\cancel{11}}$$

$$\Rightarrow (19-r)(20-r)(21-r) = 2 \times 21 \times 52$$

$$\Rightarrow (19-r)(20-r)(21-r) = 2 \times 7 \times 3 \times 4 \times$$

$$\Rightarrow (19-r)(20-r)(21-r) = 12 \times 13 \times 14$$

On Comparing, we get

$$\therefore \boxed{r = 7}$$

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Q. Prove that :-

(i.) $^{n}P_{n} = {}^{n}P_{(n-1)}$

Ans: R.H.S.

$= {}^{n}P_{(n-1)} = \frac{n!}{(n - n + 1)!} = n! = {}^{n}P_{n}$

$= L.H.S.$ Hence, L.H.S. = R.H.S.

(ii.) $^{n}P_{r} = n \cdot {}^{n-1}P_{r-1}$

Ans: L.H.S.

$= {}^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{n \cdot (n-1)!}{[(n-1) - (r-1)]!}$

$= n \cdot {}^{n-1}P_{r-1}$

$= R.H.S.$ Hence, L.H.S. = R.H.S.

(P.T.O.)

Q. Prove that :-

$${}^{n-1}P_r + r\cdot {}^{n-1}P_{r-1} = {}^{n}P_r$$

Ans: L.H.S.

$$= \left\{ \frac{(n-1)!}{(n-1-r)!} + r\cdot \frac{(n-1)!}{(n-1-r+1)!} \right\}$$

$$= \left\{ \frac{(n-1)!}{(n-1-r)!} + r\cdot \frac{(n-1)!}{(n-r)!} \right\}$$

$$= \left\{ \frac{(n-1)!}{(n-1-r)!} + r\cdot \frac{(n-1)!}{(n-r)\cdot(n-r-1)!} \right\}$$

$$= \frac{(n-1)!}{(n-r-1)!} \left\{ 1 + \frac{r}{(n-r)} \right\}$$

$$= \frac{(n-1)!}{(n-r-1)!} \left\{ \frac{n-\cancel{r}+\cancel{r}}{(n-r)} \right\}$$

$$= \frac{n\cdot(n-1)!}{(n-r)\cdot(n-r-1)!} = \frac{n!}{(n-r)!} = {}^{n}P_r$$

$$= \text{R.H.S.}$$

$\therefore$ L.H.S. = R.H.S.

Q. If $^{n}P_5 = 20 \times {^{n}P_3}$, find n?

Ans: $\Rightarrow n(n-1)(n-2)(n-3)(n-4) = 20 \times n \times (n-1) \times (n-2)$

$\Rightarrow (n-3) \times (n-4) = 5 \times 4.$

On comparing, we get

$\therefore n = 8$

Q. If $16 \times {^{n}P_3} = 13 \times {^{n+1}P_3}$, find n?

Ans: $\Rightarrow 16 \times n \times (n-1) \times (n-2) = 13 \times (n+1) \times n(n-1)$

$\Rightarrow \frac{(n-2)}{(n+1)} = \frac{13}{16}$

$\Rightarrow 16n - 32 = 13n + 13$

$\Rightarrow 3n = 13 + 32$

$\Rightarrow 3n = 45$

$\Rightarrow n = \frac{45}{3} = 15$

$\therefore n = 15$

Q If ${}^{2n}P_3 = 100 \times {}^{n}P_2$, find 'n'.

Ans:

$$\Rightarrow \frac{2n!}{(2n-3)!} = 100 \times \frac{n!}{(n-2)!}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} = 100 \times \frac{n(n-1)(n-2)!}{(n-2)!}$$

$$\Rightarrow 4(2n-1) = 100$$

$$\Rightarrow 8n-4 = 100$$

$$\Rightarrow 8n = 104$$

$$\Rightarrow n = \frac{104}{8} = 13$$

$\therefore n = 13$

Q. If ${}^{5}P_r = 2 \times {}^{6}P_{r-1}$, find 'r'?

Ans:

$$\Rightarrow \frac{5!}{(5-r)!} = 2 \times \frac{6!}{(6-r+1)!}$$

$$\Rightarrow \frac{5!}{(5-r)!} = 2 \times \frac{6!}{(7-r)!}$$

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Q. If $^{20}P_r = 13 \times {}^{20}P_{r-1}$, find 'r'.

Ans:

$\Rightarrow \frac{20!}{(20-r)!} = 13 \times \frac{20!}{(20-r+1)!}$

$\Rightarrow \frac{\cancel{20!}}{(20-r)!} = 13 \times \frac{\cancel{20!}}{(21-r)!}$

$\Rightarrow \frac{(21-r) \times \cancel{(20-r)!}}{\cancel{(20-r)!}} = 13$

$\Rightarrow 21 - r = 13$

$\Rightarrow -r = 13 - 21$

$\Rightarrow \cancel{-}r = \cancel{-}8$

$\therefore \boxed{r = 8}$

(P.T.O.)

Q. If ${}^{11}P_r = {}^{12}P_{r-1}$, find 'r'.

Ans:

$$\Rightarrow \frac{11!}{(11-r)!} = \frac{12!}{(12-r+1)!}$$

$$\Rightarrow \frac{11!}{(11-r)!} = \frac{12!}{(13-r)!}$$

$$\Rightarrow \frac{(13-r)\times(12-r)\times(11-r)!}{(11-r)!} = \frac{12\times 11!}{11!}$$

$$\Rightarrow (13-r)\times(12-r) = 4\times 3$$

On comparing, we get

$13-r=4$	$12-r=3$
$\Rightarrow r=13-4$	$\Rightarrow 12-3=r$
$\Rightarrow r=9$	$r=9$
$r=9$	

Therefore,

$\therefore$ $\boxed{r=9}$

Q. If ${}^nP_4 : {}^nP_5 = 1 : 2$, find 'n'?

Ans:

$$\frac{{}^nP_4}{{}^nP_5} = \frac{1}{2}$$

$$\Rightarrow \frac{\frac{n!}{(n-4)!}}{\frac{n!}{(n-5)!}} = \frac{1}{2}$$

$$\Rightarrow \frac{n! \times (n-5)!}{n! \times (n-4)!} = \frac{1}{2}$$

$$\Rightarrow \frac{(n-5)(n-4)!}{(n-4)!} = \frac{1}{2}$$

$$\Rightarrow 2n - 10 = 1$$

$$\Rightarrow 2n = 11$$

$$\Rightarrow \frac{(n-5)!}{(n-4)(n-5)!} = \frac{1}{2}$$

$$\Rightarrow n - 4 = 2 \quad \therefore n = 6$$

Q. If ${}^{m-1}P_3 : {}^{m+1}P_3 = 5:12$, find 'n'.

Ans:

$$\frac{{}^{m-1}P_3}{{}^{m+1}P_3} = \frac{5}{12}$$

$$\Rightarrow \frac{\frac{(m-1)!}{(m-1-3)!}}{\frac{(m+1)!}{(m+1-3)!}} = \frac{5}{12}$$

$$\Rightarrow \frac{(m-1)! \times (m-2)!}{(m+1)! \times (m-4)!} = \frac{5}{12}$$

$$\Rightarrow \frac{(m-1)! \times (m-2) \times (m-3) \times (m-4)!}{(m+1) \times m \times (m-1)! \times (m-4)!} = \frac{5}{12}$$

$$\Rightarrow 12(m-2) \times (m-3) = 5 \times m \times (n+1)$$

$$\Rightarrow 12[m^2 - 3n - 2n + 6] = 5[n^2 + n]$$

$$\Rightarrow 12[n^2 - 5n + 6] = 5n^2 + 5n$$

$$\Rightarrow 12n^2 - 60n + 72 = 5n^2 + 5n$$

$$\Rightarrow 7n^2 - 65n + 72 = 0$$

Q. If ${}^{15}P_{r-1} : {}^{16}P_{r-2} = 3 : 4$, find 'r' ?

Ans:

$\because {}^{n}P_r = \frac{n!}{(n-r)!}$

$${}^{15}P_{r-1} = \frac{15!}{[15-(r-1)]!} = \frac{15!}{[15-r+1]!}$$

$$= \frac{15!}{(16-r)!}$$

$${}^{16}P_{r-2} = \frac{16!}{(16-r+2)!} = \frac{16!}{(18-r)!}$$

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Now,

$$\Rightarrow \frac{\frac{15!}{(16-r)!}}{\frac{16!}{(18-r)!}} = \frac{3}{4}$$

$$\Rightarrow \frac{15! \times (18-r)!}{16! \times (16-r)!} = \frac{3}{4}$$

$$\Rightarrow \frac{\cancel{15!}}{\cancel{15!} \times 16} \times \frac{(18-r)(17-r)\cancel{(16-r)!}}{\cancel{(16-r)!}} = \frac{3}{4}$$

$$\Rightarrow (18-r)\cdot(17-r) = \frac{3}{\cancel{4}} \times \cancel{16}\,4$$

$$\Rightarrow (18-r)(17-r) = 4 \times 3$$

On comparing, we get

$18-r=4$	$17-r=3$
$\Rightarrow -r=4-18$	$-r=3-17$
$\Rightarrow -r=-14$	$\cancel{-}r=\cancel{-}14$
$r=14$	$r=14$

$$\therefore \boxed{r=14}$$

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Q. If ${}^{2n-1}P_n : {}^{2n+1}P_{n-1} = 22 : 7$, find 'n'.

Ans:

$$\Rightarrow \frac{{}^{2n-1}P_n}{{}^{2n+1}P_{(n-1)}} = \frac{22}{7}$$

$$\Rightarrow \frac{\frac{(2n-1)!}{(2n-1-n)!}}{\frac{(2n+1)!}{(2n+1-n+1)!}} = \frac{22}{7}$$

$$\Rightarrow \frac{(2n-1)! \times (n+2)!}{(2n+1)! \times (n-1)!} = \frac{22}{7}$$

$$\Rightarrow \frac{(2n-1)! \times (n+2) \cdot (n+1) \cdot n \, (n-1)!}{(n-1)! \times (2n+1)(2n)\,(2n-1)!} = \frac{22}{7}$$

$$\Rightarrow \frac{(n+2) \cdot (n+1) \cdot n}{(2n+1)\, 2 \times n} = \frac{22}{7}$$

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$\Rightarrow 7(n+2)(n+1) = 44(2n+1)$

$\Rightarrow 7[n^2+3n+2] = 88n+44$

$\Rightarrow 7n^2+21n+14 = 88n+44$

$\Rightarrow 7n^2+21n-88n+14-44 = 0$

$\Rightarrow 7n^2-67n-30 = 0$

$$n = \frac{67 \pm \sqrt{(67)^2 - 4\times7\times(-30)}}{2\times7}$$

$$= \frac{67 \pm \sqrt{4489+840}}{14}$$

$$= \frac{67 \pm \sqrt{5329}}{14}$$

$$= \frac{67 \pm 73}{14}$$

$\frac{67+73}{14}$ $\qquad$ $\frac{67-73}{14}$

$= \frac{140}{14} = 10$ ✓ $\qquad$ $= \frac{-6}{14} = -\frac{3}{7}$ ✗

$\therefore n = 10$

Q find n, if

$$^{n+5}P_{n+1} = \frac{11}{2}(n-1) \cdot {}^{n+3}P_n$$

Ans: $\left\{ \because {}^nP_r = \frac{n!}{(n-r)!} \right\}$

$$\Rightarrow \frac{(n+5)!}{(n+5-n-1)!} = \frac{11}{2} \times (n-1) \times \frac{(n+3)!}{(n+3-n)!}$$

$$\Rightarrow \frac{(n+5)!}{4!} = \frac{11}{2} \times (n-1) \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \frac{(n+5)! \times 3!}{4 \times 3!} = \frac{11}{2} \times (n-1) \times (n+3)!$$

$$\Rightarrow \frac{(n+5)(n+4)(n+3)!}{(n+3)!\ 4} = \frac{11}{2}(n-1)$$

$$\Rightarrow (n+5)(n+4) = 22n - 22$$

$$\Rightarrow n^2 + 9n + 20 = 22n - 22$$

$\Rightarrow n^2 + 9n - 22n + 20 + 22 = 0$

$\Rightarrow n^2 - 13n + 42 = 0$

$\Rightarrow n^2 - 6n - 7n + 42 = 0$

$\Rightarrow n(n-6) - 7(n-6) = 0$

$\Rightarrow (n-7)(n-6) = 0$

$n = 7$ or 6

Permutations

Permutations is again divided into different topics :-

Topic 1 : Permutations of 'n' different objects

→ "मतलब यह कि जिन objects को arrange करना हो उसमे से कोई भी object same नही होना चाहिए।"

इस topic मे हमारे पास चार formulae है तो कोई भी question आएगा Permutation of 'n' different objects पे based तो यह चार मे से कोई एक formulae लगा कर solve कर सकते है।

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Permutation of 'n' different objects

	Repetition not allowed	Repetition allowed
Taken all at a time	$n!$	$(n)^n$
Taken 'r' at a time	^nP_r or $^nC_r \cdot \lfloor r$	$(n)^r$

Q. In how many ways can 6 persons stand in a queue?

Ans: Reqd no. of ways =

No. of arrangements of 6 diff. objects taking all at a time

$= {}^6P_6 = (6 \times 5 \times 4 \times 3 \times 2 \times 1)$

$= 720$

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Q. How many different signals can be made by 4 flags from 6 flags, of different colours?

Ans: Required no. of signals =

no. of arrangements of 6 flags taken 4 at a time.

$= {}^6P_4 = (6 \times 5 \times 4 \times 3)$
$= 30 \times 12$
$= 360$

Q Eight athletes are participating in a race. In how many ways can the first 3 prizes be won by them?

Ans: Total no. of ways of winning first 3 prizes by 8 athletes

= no. of arrangement of 8 athletes taken 3 at a time

$= {}^8P_3 = (8 \times 7 \times 6) = 336$

Q. In how many ways can 3 different rings be worn in 4 fingers with atmost one in each finger?

Ans: Required no. of ways

= no. of ways of arranging 4 fingers taken 3 at a time.

$= {}^4P_3 = 4 \times 3 \times 2$

$= 24.$

Q. In how many ways can 5 persons occupy 3 vacant seats?

Ans: Required no. of ways

= no. of ways of arranging 5 persons taken 3 at a time.

$= {}^5P_3 = 5 \times 4 \times 3$

$= 60$ ways

(P.T.O.)

Q. In how many ways can 7 people line up at a ticket window of a cinema hall?

Ans: Required no. of ways =

No. of ~~7d~~ arrangements of 6 different objects taken all at a time.

$= {}^{7}P_{7} = (7 \times 6 \times 5 \times 4 \times 3 \times 2)$

$= 42 \times 20 \times 6$

$= 42 \times 12 \times 10$

$= 5040$ ways

Q. In how many ways can 5 children stand in a queue?

Ans: Required no. of ways =

No. of arrangements of 5 different objects taken all at a time

$= {}^{5}P_{5} = 5 \times 4 \times 3 \times 2 \times 1$

$= 20 \times 6 = 120$ ways

Q. In how many ways can 6 women draw water from 6 wells, if no well remains unused?

Ans: No. of ways =

No. of arrangements of 6 different objects taken all at a time

$= {}^6P_6 = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 30 \times 12 \times 2$

$= 24 \times 30 = 720$ ways

Q. In how many ways can 4 different books, one each in chemistry, physics, biology and mathematics, be arranged on a shelf?

Ans: No. of ways =

No. of arrangements of 4 different objects taken all at a time

$= {}^4P_4 = (4 \times 3 \times 2 \times 1)$

$= 6 \times 4 = 24.$

Q. Six students are contesting the election for the presidentship of the students' union. In how many ways can their names be listed on the ballot papers?

Ans: No. of ways in their names be listed on the ballot papers

= no. of arrangements of 6 different objects taken all at a time.

$= {}^6P_6 = 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 30 \times 24$
$= 720$ ways.

Q. 3 men have 4 coats, 6 waist coats and 5 caps. In how many ways can they wear them.

(P.T.O.)

Ans: ज्ञात No. of ways in which 3 Men, 4 coat पहन सकते हैं।

= No. of arrangements of 4 coats taken 3 at a time.

$= {}^4P_3 = 4 \times 3 \times 2$

$= 6 \times 4 = 24$ ways

ज्ञात No. of ways in which 3 Men, 6 waist coat पहन सकते हैं।

= No. of arrangements of 6 waistcoats taken 3 at a time

$= {}^6P_3 = 6 \times 5 \times 4$

$= 20 \times 6 = 120$ ways

ज्ञात no. of ways in which 3 Men, 5 cap पहन सकते हैं।

= No. of arrangements of 5 caps taken 3 at a time

$= {}^5P_3 = 5 \times 4 \times 3 = 60$ ways

Hence,

Required no. of ways in which they can wear them

$= (24 \times 120 \times 60)$
$= 72 \times 24 \times 100$
$= 172800$

Q. How many different signals can be given by using any no. of flags from 4 flags of different colours?

Ans: देखो यह signals बनाया जा सकता है 1 flag या 2 flag या 3 flag या 4 flag at a time.

No. of signal made by 4 flags

→ taking 1 at a time $= {}^4P_1 = 4$

→ taking 2 at a time $= {}^4P_2 = 4 \times 3 = 12$

→ taking 3 at a time = $^4P_3 = 4 \times 3 \times 2$
$= 6 \times 4$
$= 24$.

→ taking 4 at a time = $^4P_4 = 4 \times 3 \times 2 \times 1$
$= 6 \times 4$
$= 24$.

Hence,

∴ Total number of signals
$= (4 + 12 + 24 + 24)$
$= 64$ ways.

Q. A code is to consist of two distinct letters followed by two distinct numbers between 1 and 9.

(i) How many such codes are there?
(ii) How many of them end with an even integer?

(P.T.O.)

Ans:

(i) how many such codes are there

logic = (choosing 2 letters out of 26) × (choosing 2 no.'s out of 9)

Now,

No. of ways of choosing 2 letters out of 26

$= {}^{26}P_2 = 26 \times 25$

$= 650$ ways

&

No. of ways of choosing 2 letters out of 9

$= {}^{9}P_2 = 9 \times 8$

$= 72$ ways

Hence,

∴ Total no. of codes $= (650 \times 72)$

$= 46800$

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(ii) how many of them end with an even integer
इस case मे extreme right position मे कोई
भी numbers 2, 4, 6, & 8 मे से डाल
सकते है। तो यह position can be filled
in 4 ways.

अब अगला left position बचे हुए 8 No.s
से भर सकते इसलिए 8 तरीके है
इसको भरने का।

No. of ways of choosing pairs of even numbers.
$= (4 \times 8) = 32$

No. of ways of choosing 2 letters out of 26
$= {}^{26}P_2 = (26 \times 25) = 650$

logic = (No. of ways of choosing pair of even no.s) × (No. of ways of choosing 2 letters out of 26)

$= 32 \times 650$
$= 20800$ ways

Q. How many words (with or without meaning) can be made from the letters of the word 'MONDAY', assuming that no letter is repeated, when:

(i) 4 letters are used at a time?
(ii) all letters are used at a time.
(iii) all letters are used but the first letter is a vowel?

Ans: दिया हुआ word 'MONDAY' इसमें छः words हैं जिसमें कि O and A vowels हैं।

(i) When 4 letters are used at a time

Reqd. no. of words
= No. of arrangements of 6 letters taking 4 at a time
$= {}^6P_4 = (6 \times 5 \times 4 \times 3)$
$= 360$ words

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(ii.) all letters are used at a time

Reqd no. of words
= No. of arrangements of 6 letters taking all at a time.

$= {}^6P_6 = (6 \times 5 \times 4 \times 3 \times 2 \times 1)$
$= 30 \times 12 \times 2$
$= 24 \times 30$
$= 720$ words

(iii.) all letters are used but the first letter is a vowel?

दिया हुआ word मे दो vowels है। इसलिए first letter को हम 2 vowels मे से एक को चुन सकते है।

Thus, there are two ways of choosing the first letter.

No. of ways of arranging remaining 5 letters among themselves
$= 5!$
$= (5 \times 4 \times 3 \times 2 \times 1)$
$= 120.$

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Hence,

Required No. of words
$= (2 \times 120)$
$= 240$ words

Q. (i) How many words can be formed from the letters of the word 'TRIANGLE'?

Ans: दिया हुआ word, 'TRIANGLE' मे 8 letters है।

Total no. of words formed with these 8 letters
$= {}^8P_8 = 8! = 40320$

Hence,
Required no. of words
$= 40320$

(P.T.O.)

(ii.) How many of them will begin with T and end with E?

Ans: T _ _ _ _ _ _ E

T और E को शुरुआत और अंत में fix करने के बाद बचा हुआ छह letters को arrange कर सकते खुद में $^6P_6 = 6! = 720$ ways में

Hence,
Total no. of words, each beginning with T and ending with E

= 720.

Q. How many words can be formed with the letters of the word 'ORDINATE' so that the vowels occupy odd places?

(P.T.O.)

Ans: दिया हुआ word मे 8 letters है
इसलिए; अब उसमे, 4 vowels
& 4 consonants है।

चलो हम places को odd / even से
Mark कर ले जो भराने वाला
है, as shown below :—

$\overset{1}{(\ )}\ \overset{2}{(\ )}\ \overset{3}{(\ )}\ \overset{4}{(\ )}\ \overset{5}{(\ )}\ \overset{6}{(\ )}\ \overset{7}{(\ )}\ \overset{8}{(\ )}$

क्योंकी vowels odd places को
occupy करेगा इसलिए वह
1, 3, 5, 7 places पर placed हो
सकते है।

इसलिए,
No. of ways of arranging
vowels at 4 odd places

$$= {}^4P_4 = 4! = 24$$

बचा हुआ 4 letters दिया हुआ
word का consonant है जो को
चार even places 2, 4, 6, 8 मे
arrange किया जा सकता है

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इसलिए,
No. of ways of arranging 4 consonants at 4 even places

$= {}^4P_4 = 4! = 24$

Hence,

the total no. of words in which the vowels occupy odd places

$= (24 \times 24)$
$= 576$ words.

Q. How many 3-digit numbers can be formed by using the digits 1 to 9, if no digits is repeated?

Ans: Required no. of numbers

= No. of permutations of 9 digits taking 3 at a time.

$= {}^9P_3 = (9 \times 8 \times 7) = 504.$

Q. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of them will be even?

Ans: Required No. of 4-digit no.'s

= no. of arrangements of 5 digits taking 4 at a time

$= {}^5P_4 = (5 \times 4 \times 3 \times 2)$
$= 20 \times 6$
$= 120$ numbers

(Particular case)

एक even no. के लिए unit's digit 2 या 4 होगा।

इसलिए 2 ways of filling the unit's digit

No. of ways of filling remaining 3 places with the remaining 4 digits.

$$= {}^4p_3 = (4 \times 3 \times 2)$$
$$= 12 \times 2$$
$$= 24$$

Q. How many 6-digit telephone numbers can be constructed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 if each number starts with 35 and no digit appears more than once?

Ans: 10 digits में से दो digits namely 3 & 5 are reserved for the two extreme left places of the desired no.s

3 5 _ _ _ _

बची हुआ 8 digits को बचे हुए चार position में place करने का तरीका

$$= {}^8p_4 = (8 \times 7 \times 6 \times 5)$$
$$= 1680 \text{ ways}$$

Hence, 1680 telephone no.'s can be constructed.

Q. It is required to seat 5 men & 3 women in a row so that the women occupy the even places. How many such arrangements are possible?

Ans:

1	2	3	4	5	6	7	8
( )	( )	( )	( )	( )	( )	( )	( )

तो question मे यह बोला गया है की 5 Men और 3 women को एक row मे बैठाना है ऐसे की women even place पर हो।

तो अगर हम ध्यान से देखे तो total पाँच position है woman के लिए पाँच place मे तीन woman को बैठाना है।

No. of ways of arranging 5 men in 5 places

$= 5! = 120$

Now,

No. of ways of arranging 3 women in 5 places

$= {}^5P_3 = 5 \times 4 \times 3 = 12 \times 5 = 60$

Hence,

Total such arrangements possible

$= 120 \times 60$

$= 7200$ arrangements.

Q. There are 6 items in column A and 6 items in column B. A student is asked to match each item in column A with an item in column B. How many possible, correct or incorrect answers are there to this question?

A	B
A_1	B_1
A_2	B_2
A_3	B_3
A_4	B_4
A_5	B_5
A_6	B_6

Ans:

देखो A_1 के पास 6 option है B मे Match करवाने के लिए ; इसलिए
→ 6 ways

A_2 के पास 5 option है B मे बचा Match करवाने के लिए, इसलिए
→ 5 ways

A_3 के पास 4 option है B मे बचा Match करवाने के लिए ; इसलिए
→ 4 ways.

Similarly

A_4 → 3 ways

A_5 → 2 ways

A_6 → 1 way.

∴ possible no. of correct or incorrect

answers to this question

$= (6 \times 5 \times 4 \times 3 \times 2 \times 1)$
$= 30 \times 24$
$= 720$ ways.

Q. Five letters F, K, R, T and V one in each were purchased from a plastic warehouse. How many ordered pairs of letters, to be used as initials, can be formed from them?

Ans: initials मतलब होता है नाम का Short Abbreviation
ex :- Amritesh Makarchand Tomar
A.M.T.

अब देखो हमें question मे यह दिया है कि हमने F, K, R, T & V हमने एक plastic warehouse से खरीद के लाया है अब हमे यह बताना है की कितना दो letter का initial यानी की ordered pair बनाया जा सकता है।

(P.T.O.)

= no. of ways of selecting 3 different thing out of 10 objects.

$= {}^{10}P_3 = 10 \times 9 \times 8$

$= 720$ ways.

Q. There are six periods in each working day of the school. In how many ways can one arrange 5 subjects such that each subject is allowed at least one period?

Ans: हमारे पास six period है एक working day में तो हम कितने तरीके से arrange कर सकते है 5 subjects को six periods में जबकी each subject is allowed at least once

5 subjects   6 periods

6 period में से, 5 arrange किया जा सकता है $^{5}P_5$ ways में।

(P.T.O.)

तथा 1 period may be arranged for anyone of the five subjects in $5p_1$ ways.

$\therefore$ Required No. of ways

$= ({}^5p_3 \times {}^5p_1)$.

Wrong Method

(5 Subjects)

Periods → P_1 P_2 P_3 P_3 P_4 P_5

अब देखो हमारे पास 5 subject है और इस को 6 period में arrange करना है।

Subject 1 → 6 ways
Subject 2 → 5 ways
Subject 3 → 4 ways
Subject 4 → 3 ways
Subject 5 → 2 ways

$= (6 \times 5 \times 4 \times 3 \times 2) = {}^6p_5$

अब इस logic में देखो की हर arrangement में एक period में कोई भी subject appoint नहीं हो पाएगा।

Q. In how many ways can 6 pictures be hung from 4 picture nails on a wall?

Ans. | 6 pictures | | 4 picture nails |

No. of ways in which 6 pictures be hung from 4 picture nails on a wall

= No. of ways of arranging 4 different objects out of 6 given objects

$= {}^6P_4 = 6 \times 5 \times 4 \times 3$
$= 30 \times 12$
$= 360$ ways

(P.T.O.)

Ans: No. of ways of forming 4 lettered word out of ~~digi~~ letters of the word 'NUMBERS'.

= No. of ways of arranging 7 objects taken 4 at a time.

$= {}^7P_4 = 7 \times 6 \times 5 \times 4$

$= 42 \times 20$

$= 840$ words

Q. How many words can be ~~fr~~ formed from the letters of the word 'SUNDAY'? How many of these begin with D?

Ans: 'SUNDAY' word का 6 letters है।

No. of ways/words can be formed from the letters of the word 'SUNDAY'

= No. of ways of arranging 6 objects taken all at a time.

$$= {}^6P_6 = 6!$$
$$= 6 \times 5 \times 4 \times 3 \times 2$$
$$= 6 \times 6 \times 20$$
$$= 36 \times 20$$
$$= 720 \text{ words}$$

Second Section

No. of words that begin with 'D'

D ___ ___ ___ ___ ___

अब देखो एक place already filled है और एक letter भी used है तो बचा 5 place और 5 letter

इसलिए 5 place मे 5 letters को arrange करने के तरीके

$$= {}^5P_5 = 5!$$
$$= 5 \times 4 \times 3 \times 2 \times 1$$
$$= 20 \times 6$$
$$= 120 \text{ words.}$$

Q. How many words beginning with C and ending with Y can be formed by using the letters of the word 'COURTSEY'?

Ans: C _ _ _ _ _ _ Y

अब देखो पहला और अंतिम place already filled है इसलिए बचा छह place और छह letters.

∴ No. of words that can be formed so that beginning with C and ending with Y from letters of word 'COURTSEY'.

= No. of ways of arranging 6 letters taken all at a time

$= {}^{6}P_{3}$

$= 6!$

= 720 words.

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Q. Find the number of permutations of the letters of the word 'ENGLISH'. How many of these begin with E and end with I ?

Ans:

E _ _ _ _ _ I

अब देखो extreme right & extreme left already filled है इसलिए बचा 5 places और 5 letters.

∴ No. of ways of the letters of the word ENGLISH

= No. of ways of arranging 7 objects taken at a time

$= {}^7P_7 = 7!$

$= 5040$

(P.T.O.)

Now.

No. of ways of forming words such that it begin with E & end with I.

= No. of arrangement of selecting 5 objects taken all at a time.

$= {}^5P_5 = 5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 120$ words.

In how many ways can the letters of the word 'HEXAGON' be permuted? In how many words will the vowels be together?

(P.T.O.)

Q. In how many ways can 4 letters be posted in 3 letter boxes?

Ans: हरेक letter 3 में से कोई एक letter box में post किया जा सकता है।

इसलिए, हरेक letter को 3 तरीके से post किया जा सकता है

इस कारण

∴ 4 letters को post करने के तरीके :-

$= (3 \times 3 \times 3 \times 3)$

$= 3^4 = 81$ ways.

∵ No. of permutations of 'n' different objects taken 'r' at a time when repetition of objects is allowed.

$= (n)^r$

Q. In how many ways can 6 different rings be worn in 4 fingers of the hand?

Ans: हर एक Ring को चार मे से किसी एक finger मे पहना जा सकता है

इसलिए, हरेक Ring को 4 ways मे पहना जा सकता है।

∴ 6 Rings may be worn in

$= (4 \times 4 \times 4 \times 4 \times 4 \times 4)$

$= 4^6 = 4096$ ways.

Q. In how many ways can 5 apples be distributed among 6 boys, there being no restriction to the number of apples each boy may get?

Ans: हरेक apple 6 boys मे से किसी एक को दिया जा सकता है

इसलिए, हरेक apple may given in 6 ways.

∴ 5 apples given in ~~6 ways~~

$= (6 \times 6 \times 6 \times 6 \times 6) = 6^5 = 7776$ ways

Q. In how many ways can 3 prizes be distributed among 4 boys, when
(i.) no boy gets more than 1 prize;
(ii.) a boy may get any no. of prizes;
(iii.) no boy gets all the prizes?

Ans: (i.) When no boy gets more than 1 prize

First prize को हम किसी भी 4 boys में से एक को दे सकते हैं इसलिए, → 4 ways

Second prize को हम 3 boys में से किसी एक को दे सकते हैं, क्योंकि जिस boy को 1st prize मिला नहीं सकती। → 3 ways

third prize को हम 2 boys में से किसी एक को दे सकते हैं
→ 2 ways

∴ Required no. of ways
$= (4 \times 3 \times 2) = 24$ ways.

(ii) a boy may get any no. of prizes

First prize को 4 boys में से किसी एक को दिया जा सकता है in → 4 ways

क्योंकि एक boy को ~~किसी~~ कितनी भी ~~no. के~~ prizes मिल सकती हैं ; इसलिए

Second prize को 4 boys में से किसी एक को दिया जा सकता है → 4 ways.

Similarly,

third prize को 4 ways.

Hence

Required number of ways

$= (4 \times 4 \times 4)$

$= 4^3 = 64$ ways.

(iii.) When no boy gets all the prizes:

No. of ways in which एक boy को सभी prize मिल जाएँ is 4, क्योंकि 4 में से कोई भी एक को सभी prizes मिल जाएगी।

Hence,

No. of ways in which a boy does not get all the prizes

$= (64-4) = 60$ ways.

Q. How many four-digit numbers can be formed with the digits 1, 2, 3, 4, 5, 6 when a digit may be repeated any number of times in any arrangement?

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Ans: साफ है कि thousand's place को fill करने के 6 ways हैं क्योंकि 6 digits में से कोई एक digit place करेंगे।

∵ कोई restriction repetition of digits को लेकर नहीं है इसलिए each one of the hundred's, ten's and unit's digits can be filled in 6 ways.

Hence,

the reqd. no. of numbers

$= (6 \times 6 \times 6)$

$= 36 \times 6$

$= 216$ numbers.

Q. How many 4-digit numbers are there when a digit may be repeated any number of times?

(P.T.O.)

Ans: □ □ □ □

साफ है कि 0 thousand's place पर place नहीं किया जा सकता है, क्योंकी यह place को 1 to 9 मे से कोई एक digit से भरा जा सकता है।

Thus, there are 9 ways of filling the thousand's place

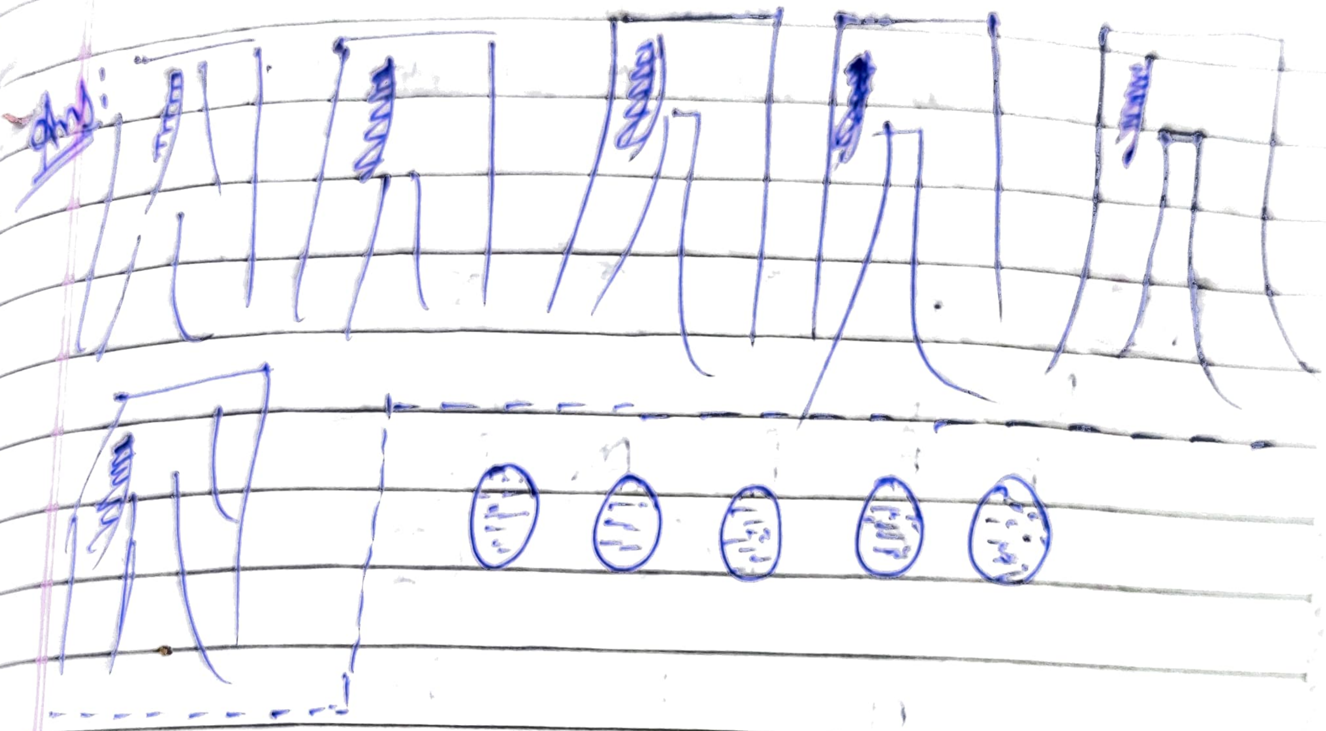
Since repetition of digits is allowed, हरेक बचा हुआ 3 place को 10 each ways से भरा जा सकता है i.e. with any digit from 0 to 9.

So, the required No. of numbers

$= (9 \times 10^3) = 9000$ Num

Q. A child has 6 pockets. In how many ways can he put 5 marbles in his pocket?

Ans:



~~पहला pocket में 5 Marble~~

पहला Marble के पास छह pocket का option — 6 ways.

दूसरा Marble के पास छह pocket का option — 6 ways

Similarly

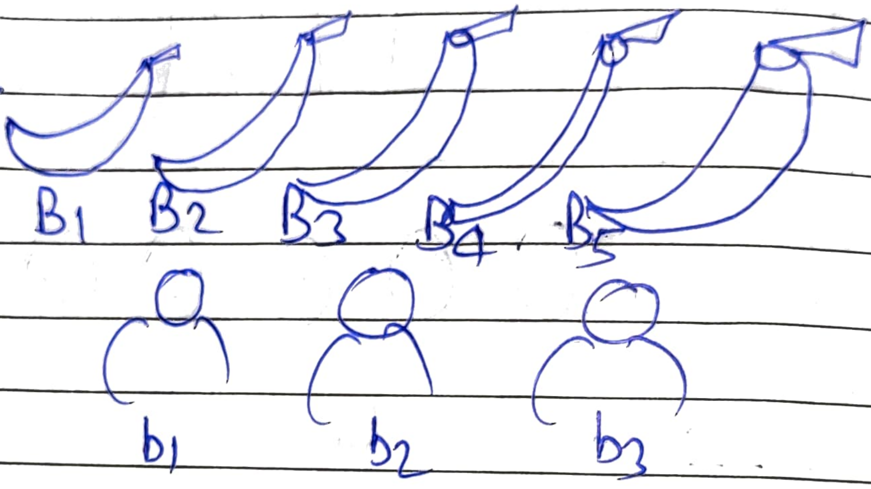
तीसरा, चौथा और पाँचवा 6 ways each

$= 6 \times 6 \times 6 \times 6 \times 6$

$= 6^5$ ways

Q In how many ways can 5 bananas be distributed among 3 boys, there being no restriction to the number of bananas each boy may get?

Ans:



पहला banana B_1 को हम 3 में से किसी एक boy को दे सकते हैं → 3 ways

& दूसरा banana B_2 को हम 3 में से किसी एक boy को दे सकते हैं → 3 ways

& तीसरा banana B_3 को हम 3 में से किसी एक boy को दे सकते हैं → 3 ways

& चौथा banana B_4 को हम 3 में से किसी एक boy को दे सकते हैं → 3 ways

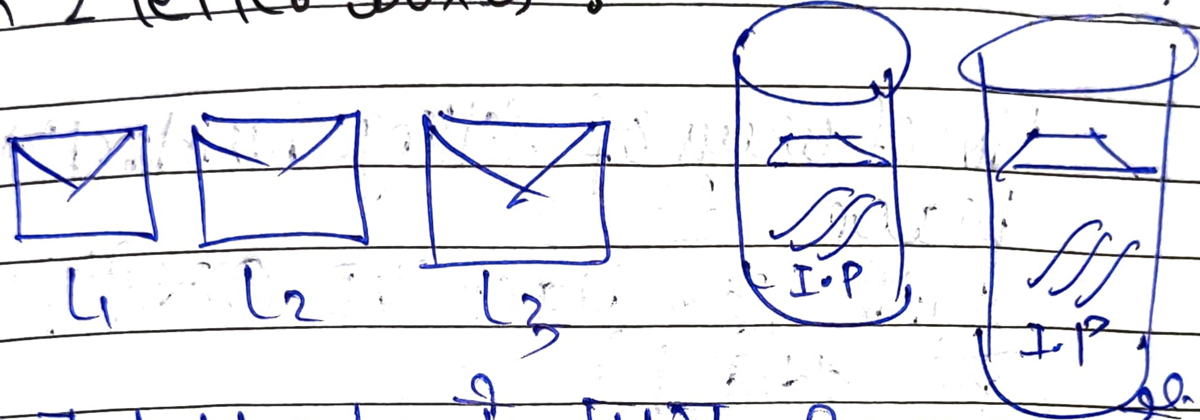
& पाँचवा banana B_5 को हम 3 में से किसी एक boy को दे सकते हैं → 3 ways

$= 3 \times 3 \times 3 \times 3 \times 3$

$= 3^5$ ways.

Q. In how many ways can 3 letters be posted in 2 letter boxes?

Ans:



पहला letter L_1 के पास 2 way है letter को post करने के लिए → 2 ways

(2) दूसरा letter L_2 के पास 2 way है letter को post करने के लिए → 2 ways

(3) तीसरा letter L_3 के पास 2 way है letter को post करने के लिए → 2 ways.

∴ No. of ways in which 3 letters be posted in 2 letter boxes

$= (2 \times 2 \times 2)$

$= 8$ ways.

Q. How many 3-digit numbers are there when a digit may be repeated any no. of times?

Ans: □ □ □

No. of ways of filling hundred's place = 9
(क्योंकि इस place पर 0 नहीं place कर सकते।)

& No. of ways of filling ten's place = 10.

& No. of ways of filling unit's place = 10.

Hence, Total no. of 3-digit numbers when a digit may be repeated any no. of times

$= 9 \times 10 \times 10$

$= 900$ ways

Q. How many 4-digit numbers can be formed with the digits 0, 2, 3, 4, 5 when a digit may be repeated any number of times in any arrangement?

Ans: digits available → 0, 2, 3, 4, 5

□ □ □ □

No. of ways of filling thousand's place = ~~9~~ 4
(क्योंकी इस place पर 0 नहीं place कर सकतें।) → 9 ways.

&

No. of ways of filling hundred's place = ~~10~~ 5

& No. of ways of filling ten's place = ~~10~~ 5

& No. of ways of filling unit's place = 5 ~~10~~

Hence, Total no. of 4-digit no.s formed with digits 0, 2, 3, 4 & 5

$= (4 \times 5 \times 5 \times 5)$

$= 500$ numbers

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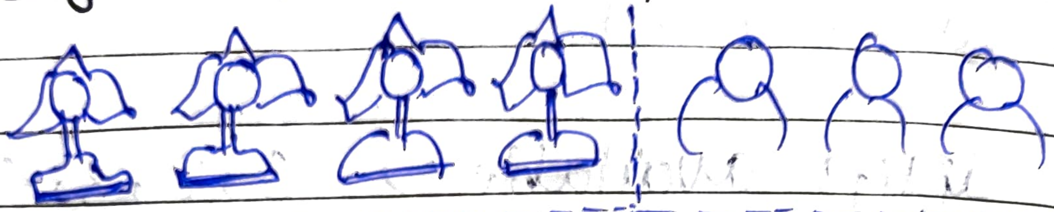
Q In how many can 4 prizes be given to 3 boys when a boy is eligible for all prizes?

Ans: 

पहला prize को तीन बच्चो में से किसी एक को देंगे i.e. 3 ways.

& दूसरा prize को तीन बच्चो में से किसी एक को देंगे i.e. 3 ways

& तीसरा prize को तीन बच्चो में से किसी एक को देंगे i.e. 3 ways

& चौथा prize को तीन बच्चो में से किसी एक को देंगे i.e. 3 ways

$= (3 \times 3 \times 3 \times 3)$

$= 81$ ways

Q. There are 4 candidates for the post of a chairman and one is to be elected by votes of 5 men. In how many ways can the votes

Ans: 4 candidate(s) 5 men

पहले Man के पास 4 candidate को vote देना का option है → 4 ways

(4) दूसरे Man के पास 4 candidate को vote देना का option है → 4 ways

(4) तीसरे Man के पास 4 candidate को vote देना का option है → 4 ways

(4) चौथे Man के पास 4 candidate को vote देना का option है → 4 ways

(4) पाँचवे Man के पास 4 candidate को vote देना का option है → 4 ways.

∴ Total No. of ways in which votes be given

$= (4 \times 4 \times 4 \times 4 \times 4)$

$= 1024$ ways.

Permutations of Objects not all different

मतलब यह की दि गई Object जिनको हम को arrange करना है उसमें से दो या उससे अधिक objects same है।

For example :-

Let's take two word "DEE" & "DOE"

No. of arrangements of "DEE"	No. of arrangements of "DOE"
DEE	DOE
EDE	DEO
EED	ODE
	OED
	EDO
	EOD
3	**6**

⇒ if you have three letters out of which 2 letters are same then you have '3' arrangements.

Let take word "EEE"

No. of arrangement possible = 1.

⇒ तो हमको यह समझ मे आया की जब तीन-lettered word मे तीनो letter distinct था तो उसके 6 arrangement possible है;

⇒ जब तीन lettered word मे दो letter repeated थे तो 4 arrangement possible थे & जब तीनो letter same थे तो 1 arrangement possible है।

तो यह पता लगा की जैसे No. of repetition of object to be arrange बढ़ रहा है वैसे no. of arrangement घट रहा है।

तो ऐसे count करना थोड़ा difficult होगा let us try to logically see if there is any formulae to do this.

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illustration :-

एक example लेता हूँ
APPLE

अब "APPLE" मे दो letters 'P' repeated हैं अब मै इन 5 letters को 5 card assume करता हूँ

[A] [P] [P] [L] [E]   __ __ __ __ __

अब मुझे इन पाँच कार्डस को इन 5 place मे arrange करना है।

मैंने सबसे पहले card A को उठाया अब इसको मै 5 place मे कोई भी एक place मे रख सकता हूँ
so, [A] card को रखने के लिए 5 ways

इसके बाद
[L] card को उठाया, इसको place करने के लिए 4 place मे कोई भी एक place मे रख सकता हूँ।

(P.T.O.)

इसके बाद

[E] card को उठाया, इसको place करने के लिए 3 place में कोई भी एक place में रख सकता हूं।

अब समझो
अगर दो [P] card के जगह दो distinct letter होता let's say P & Q तो 2 & 1 ways respectively way of arrangement होता।
but, अब क्योंकि दो P (same) है; इसलिए 1 ही way होगा।

APPLE → E P A P L

E	P	A	P	L
5	1	4		3

इसको जब मैंने No. of arrangement निकाला "APPLE" का तो आया :-

= 5×4×3×2×1

इसमे जो दो 2 आना था जब दो letter distinct होता था तो नही आया मगर मै same चीज को ऐसे लिख सकता हूँ।

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{\lfloor 2} = \frac{\lfloor 5}{\lfloor 2}$$

⇒ अगर मान लिजिए मेरे पास 3 letters repeat हो रहे है

for example "APPL" लिया

___ ___ ___ ___ ___

अब देखो A को arrange करने का 5 तरीका, L को arrange करने का 4 तरीका, पर जब हम P पे आए तो क्योंकी तीनो P same है इसलिए 1 तरीका।

$= 5 \times 4 \times 1$

(P.T.O.)

या इसको मैं ऐसा भी लिख सकता हूँ

$$= \frac{5 \times 4 \times 3!}{3!} = \frac{5!}{3!}$$

➤ अब हमने दो तीन example देखे अब चाह रहे हैं की इन चीजों को generalize कर दें ;

जब दो letter repeat कर रहा था तो 5 objects के लिए 5! को आ रहा है ('APPLE' वाले example में) plus जितना बार letter P repeat हो रहा है वो factorial नीचे आ रहा है।

जब तीन letters repeat कर रहा था तो 5 objects के लिए 5! तो आ रहा है ('APPL' वाले example में) plus जितना बार letter P repeat हो रहा है वो उतना का factorial नीचे आ रहा है।

So, जहाँ 'n' objects are alike

$$= \frac{\text{total arrangements}}{\lfloor\underline{n}}$$

Examples :-

(i) RRAPPLE $\rightarrow \frac{7!}{2! \cdot 2!}$

(ii) AADEED $\rightarrow \frac{6!}{2! \cdot 2! \cdot 2!}$

(iii) AGREE $\rightarrow \frac{5!}{2!}$

(iv) INDIA $\rightarrow \frac{5!}{2!}$

(v) MISSISSIPPI $\rightarrow \frac{11!}{4! \cdot 4! \cdot 2!}$

(vi) INDEPENDENCE

$\hookrightarrow \frac{12!}{3! \cdot 2! \cdot 4!}$

Restricted Permutation

If some objects are to be together.

let say for example

'TRIANGLE' — it has 8 letters and so it can be arranged in 8! ways.

अब हमने बोला की Find No. of arrangements when vowels are together तो जब भी तुम्हें यह word मिले "together" so this is the standard word जो आपको identify करना है question में

तो जब भी मुझे यह word 'together' मिले we will use method called "String Method"

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String Method

(T), (R), (N), (G), (L) (I, A, E) → $6! \times 3!$

$6!$ → external arrangement

$3!$ → internal arrangement

देखो! हमे क्या करना है no. of ways find करना है जब all vowels in word "TRIANGLE" are together तो हमने क्या किया IAE को एक string से बाँध लिया मतलब अब (I, A & E) एक object consider करेंगे

तो अब (T), (R), (N), (G), (L) & (IAE)

यानी 6 object, को arrange करने के 6! ways इसको हम external arrangement बोलेंगे।
अब देखो अब हमने external arrangement तो 6! ways मे कर दिया है।

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(मगर) अब ध्यान दो हमने जो तीन letters को बाँधा था तो भी तो उस box में अंदर arrange होंगे; इसलिए उनका arrangement के लिए भी 3! ways होगा (3!)

Some instance of external arrangement	Some instance of internal arrangement
T (IAE) RNGL	IAE
	IEA
RTN (IAE) GL	EAI
	EIA
(IAE) NRTGL	AEI
⋮	AIE

★ अब question है की all vowels did not occur together.

तो इसके लिए :—

CONCEPT

No. of arrangements when all vowels did not occur together.

= (Total arrangements) − (No. of arrangements when all vowels occur together)

$= 8! - (6! \times 3!)$

Gap Method

⇒ "इसमे keyword है: No two objects come together."

example लिया word "TRIANGLE" को

इसमे consonants	इसमे vowels
T	I
R	A
N	E
G	
L	

इसमे question है की 'find the no. of ways of arranging the letters of the word "TRIANGLE' when no two vowels are together.'

_ T _ R _ N _ G _ L _

इस question को solve करने के लिए
8! − (6! × 3!) नहीं कर सकते
i.e. No. of ways when all vowels did not come together.

इसके लिए कुछ और logic apply करेंगे।

हमने पाँच vowels को alternate positions (एक place दो letters के बीच खाली रखते हुए) में arrange कर दिया
तो no. of ways of arranging 5 letters in 5 places = 5!

अब मैं क्या करता हूँ यह पाँच students के बीच में we will calculate the gaps, तो calculate करने पर 6 gaps आया।

यह 6 gaps में कोई भी तीन gaps में अगर में तीनों letters को place किया तो, obviously no two vowels will be together.

ex:

अगर मान लो यह configuration है:

Ⓘ T O R O N Ⓐ G O L Ⓔ

I और A के बीच मे T, R & N
& A और E के बीच मे G & L

तो यह हो मैं कर रहा हूँ
तो 6 gaps मे से 3 letters को कैसे place कर सकता हूँ i.e. in $^6P_3$ ways.

Hence,

No. of ways of arranging letters of word "TRIANGLE"

$= 5! \times {}^6P_3$

$= 120 \times 6 \times 5 \times 4$
$= 24 \times 5 \times 120$
$= 120 \times 120$
$= 14400$ ways

Q. Find the no. of ways in which 5 boys and 3 girls can be arranged in a row so that no two girls are together.

Ans: अब समझो यहाँ gap Method लगाएंगे

① 5 boys को 5 alternate place में खड़ा करने के 5! ways

② बचे हुए 6 place में 3 girls को खड़ा करने के $^6P_3$ ways.

Hence,

Total no. of ways in which 5 boys and 3 girls can be arranged in a row so that no two girls are together

$= 5! \times {}^6P_3 = 120 \times 120 = 14400$

G O B O B O B O B O B O

Q. There are 2 English, 3 sanskrit and 4 Hindi books. In how many ways can they be arranged on a shelf so as to keep all the books of the same language together?

Ans: यहाँ String Method लगाएंगे

→ 2 English बुक्स का एक packet बना लिया

→ 3 sanskrit बुक्स का एक packet बना लिया

→ 4 Hindi बुक्स का एक packet बना लिया

अब हमारे पास सिर्फ तीन packet बचे

अब देखो external arrangement :-

No. of ways of arranging the 3 packets $= {}^3P_3 = 3! = 6$

Now, internal arrangement :-

No. of ways of arranging 2 English books
$= 2! = 2$.

No. of ways of arranging 3 Sanskrit books
$= 3! = 6$.

No. of ways of arranging 4 Hindi books
$= 4! = 24$.

Hence,

the required number of ways
$= (2 \times 6 \times 24) \times 6$
$= 1728$ ways

(P.T.O.)

Q. In how many ways can 6 balls of different colours, namely black, white, blue, red, green and yellow be arranged in a row in such a way that the black and white balls are never together?

Ans: black ball और white ball को बांध दिया और (black & white) को एक ball मान लिया।

अब (black & white) और 4 other balls may be arranged in

$= 5P_5 = 5! = 120$ ways

Also, यह दो बॉल black & white can be arranged among themselves in $2! = 2$ ways

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∴ Total no. of arrangements with black and white balls together

$= (120 \times 2) = 240$

Hence,

Numbers of ways of arranging 6 balls among themselves

$= {}^6P_6 = 6! = 720$

Now,

Number of ways of arranging 6 balls such that black & white balls are never together

$= (720 - 240)$

$= 480$

Q. The principal wants to arrange 5 students on a platform such that the boy Sajal occupies the second position and the girl Tanvy is always adjacent to the girl Aditi. How many such arrangements are possible?

Ans: Let the five seats be arranged as shown below

I	II	III	IV	V
□	S	□	□	□

Sajal को second position में fixed कर दिया।

क्योंकि Tanvy और Aditi को साथ में बैठना है; इसलिए first seat को ये भी नहीं occupy कर सकती है।

पहला seat can be occupied by any of the two remaining students in $2! =$ ways

No. of ways in which 2 seats namely III, IV (or) IV, V maybe occupied by Tanvy and Aditi

$$= (2!) + (2!) = (2+2) = 4$$

बचा हुआ seat, 5th student occupy कर सकता है 1 way में।

Hence, the required no. of arrangements

$$= (2 \times 4 \times 1)$$
$$= 8.$$

Q. How many words with or without meaning can be formed from the letters of the word 'DAUGHTER', so that

(i) all vowels occur together?
(ii) all vowels do not occur together?

Ans: दिया हुआ word 'DAUGHTER' मे 3 vowels A, U, E और 5 consonants D, G, H, T, R.

Case (i) when all vowels occur together

(A, U, E) को एक single letter मान लिया।
फिर, this letter (A, U, E) along with 5 other letters can be arranged in

$^6P_6 = 6!$ ways
$= (6 \times 5 \times 4 \times 3 \times 2 \times 1)$ ways
$= 30 \times 12 \times 2$
$= 24 \times 30$
$= 720$ ways.

3 vowels को खुद मे arrange करने के $3! = 6$ ways.

Hence,
the required no. of words with vowels together $= (6!) \times (3!)$

$=(720 \times 6) = 4320.$

Case (ii) When all vowels do not occur together

No. of words जो दि गई word के 8 letters के द्वारा बनाया जा सकता है

$= {}^{8}P_{8} = 8! = (8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)$

$= 40320.$

Hence;

Number of words in which all ~~word~~ vowels are never together.

= (total no. of words) − (no. of words with all vowels together)

$= (40320 - 4320)$

$= 36000$

Q. In how many ways can the letters of the word 'HEXAGON' be permuted? In how many words will the vowels be together?

Ans:

Consonants: H, X, G, N

Vowels: E, A, O

Let's apply **String Method**

E, A, & O को एक thread से बाँध दिया तो अब हमारे पास 5 letter तथा H, X, G, N & (E, A, O)

No. of ways of arranging letters of the word 'HEXAGON'

$= 7! = 7!$

$= 5040$ ways

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* words in which vowels come together

= No. of ways of arranging 5 letters × No. of ways of arranging three letters

$= 5! \times 3!$

$= 120 \times 6$

$= 720$ ways.

Q. How many ways can the letters of the word 'FAILURE' be arranged so that the consonants may occupy only odd positions?

Ans: दिया हुआ word 'FAILURE' के पास 3 consonants & 4 vowels हैं जो कि seven places में arrange किया जाएगा, shown below:-

(1) (2) (3) (4) (5) (6) (7)

अब, 3 consonants को हम 4 marked places i.e. 1, 3, 5, 7 में से किसी भी 3 places में रख सकते हैं

Number of ways of arrang. the consonants

= No. of arrangement of 4 different objects taken 3 at a time.

$= {}^4P_3 = 24.$

(8), 4 बचे हुए vowels बचे हुए 4 places में (namely 1, 3, 5, 7) arrange होंगे in

${}^4P_4 = 4! = 24$ ways

∴ Require number of ways.

= (No. of ways of arranging the consonants) × (No. of ways of arranging the vowels)

$= 24 \times 24$

$= 576$ ways.

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Q. How many words can be formed out of the letters of the word 'ORIENTAL' so that the vowels always occupy the odd places?

Ans: दिया हुआ word 'ORIENTAL' में 4 vowels & 4 consonants हैं।

Let us fix the 8 places as shown below:

(1) (2) (3) (4) (5) (6) (7) (8)

4 vowels को 4 places में arrange करने के ways (namely 1, 3, 5, 7)

$= {}^4P_4 = 4! = 24$ ways.

(4) 4 consonants को 4 places (namely 2, 4, 6, 8) in

$= {}^4P_4 = 24$ ways.

∴ Required no. of ways

$= (24 \times 24) = 576$ ways

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Q. Find the no. of different permutations of the letters of the word, 'BANANA'.

Ans: देखो BANANA में 6 letters हैं जिसमें three are alike of one kind (3 A's), two are alike of second kind (2 N's) and one of its own kind (1 B).

∴ Total no. of their permutations

$$= \frac{6!}{(3!)\times(2!)\times(1!)} = 60$$

Q. How many words can be formed using the letter A thrice, the letter B twice & the letter C thrice?

(P.T.O.)

Ans: हमें 8 letters दिया हुआ है namely
A, A, A, B, B, C, C, C.
इन letters में तीन एक तरह के हैं (3 A's) ; two एक तरह के है (2 B's) और तीन एक तरह के है (3 C's).

Hence,
the required no. of permutations

$$= \frac{8!}{(3!) \times (2!) \times (3!)} = 560 \text{ words.}$$

Q. (i) Find how many arrangements can be made with the letters of the word 'MATHEMATICS'.

(ii) In how many of them are the vowels together?

Ans: (i) ग्यारह letters है word MATHEMATICS में।
जिनमें M दो बार आता है, A तीन बार आता है & T ~~तीन~~ दो बार और बाकी बचा अलग है सभी।

Hence,

∴ the total no. of arrangements of the given letters

$$= \frac{11!}{(2!)\times(2!)\times(2!)} = 4989600.$$

(ii) दिए गए word में 4 vowels namely A, E, A, I.
अब इन ~~तीनों~~ को एक साथ बांध दिया, AEEA अब एक letter के भांति मानेंगे।

अब हमें 8 letters को arrange करना है ~~जिसमें~~ MTHMTICS + AEAI जिसमें की M दो बार occur होगा, T दो बार occur होगा और बचा बाकी सभी different है।

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इसलिए,

No. of all such arrangements

$$= \frac{8!}{(2!) \times (2!)} = 10080$$

अब, 4 vowels में से A दो बार आ रहा है और बाकी सभी अलग है।

इसलिए, No. of arrangements of these vowels

$$= \frac{4!}{2!} = 12.$$

Hence, the no. of arrangements in which 4 vowels are together

$= (10080 \times 12)$

$= 120960.$

(P.T.O.)

Q. (I.) Find the number of arrangements of the letters of the word, 'INDEPENDENCE'.

(II) In how many of these arrangements :—

(i) do the words start with P?
(ii) do all the vowels occur together?
(iii) do the vowels never occur together?
(iv) do the words begin with I and end in P?

Ans: (I.) दिया हुआ word, 'INDEPENDENCE' में 12 letters है जिनमें N तीन बार आया है, E चार बार आया है, D 2 बार आया है और बाकी सभी अलग है

Hence,

Required no. of arrangements

$$= \frac{12!}{(3!) \times (4!) \times (2!)} = 1663200$$

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(II.)

(i) the words start with P.

P को extreme left position में fix करने के बाद मेरे पास 11 letters बचे जिनमें को 3 N's, 4 E's, 2 D's, 1 I and 1 C.

So, the no. of words beginning with P
= no. of arrangements of 11 letters out of which there are 3 N's, 4 E's, 2 D's, 1 I and 1 C.

$$= \frac{11!}{(3!) \times (4!) \times (2!)} = 138600.$$

(ii) all vowels occur together

हमारे पास 5 vowels है चलो इसे एक single letter [E E E E I]

consonants: N, D, P, N, D, N, C

vowels: E, E, E, E, I

थह letter with 7 other
remaining letters will give us
8 letters जिसमें की 3 N's,
2 D's, 1 P, 1 C and 1 letter
EEEEI.

No. of ~~a~~ all such arrangements

$$= \frac{8!}{(3!) \times (2!)} = 3360$$

No. of arrangements of 5 vowels, namely 4 E's and 1 I

$$= \frac{5!}{4!} = 5$$

Hence,

the no. of words in which vowels are together

$$= (3360 \times 5)$$
$$= 16800$$

(iii) vowels never occur together

No. of arrangements in which vowels do not occur together

= (total no. of arrangements) − (no. of arrangements in which vowels occur together)

$= (1663200 - 16800) = 1646400$

(iv) words begin with I and end in P?

I को extreme left में fix कर दिया और P को extreme right में fix कर दिया।
अब हमारे पास सिर्फ 10 letter बचे हैं जिसमें 3 N's, 4 E's, 2 D's and 1 C.

Hence the number of words which begin with I and end in P

$$= \frac{10!}{(3!) \times (4!) \times (2!)} = 12600$$

Q. (i) How many different words can be formed by using all the letters of the word, 'ALLAHABAD'?
In how many of them:
(ii) both L's do not come together?
(iii) the vowels occupy the even positions?

Ans: (i) दिया हुआ word 'ALLAHABAD' मे 9 letters है जिसमे 4A's, 2L's, 1H, 1B and 1D.

Hence,

The no. of different words formed by using all the letters of the given word

$$= \frac{9!}{(4!) \times (2!)} = 7560$$

(P.T.O.)

(iii) मान लेते हैं, दोनों L को एक [LL] as 1 letter

अब 8 letters, namely [LL], 4 A's, 1 H, 1 B and 1 D

इसलिए, the number of words having both L together

$$= \frac{8!}{4!} = 8 \times 5 \times 7 \times 6$$
$$= 1680$$

Hence,

No. of words with both L not occuring together

$= (7560 - 1680)$

$= 5880$ words

(P.T.O.)

(iii) यहाँ पे 4 vowels है और सभी एक जैसा है i.e., 4 A's.

Also, यहाँ पे 4 even places, namely 2nd, 4th, 6th and 8th.

इसलिए अब A को हम ऊपर mentioned even places में place कर दिए, जैसा की नीचे दिखाया गया है।

□ A □ A □ A □ A □

No. of arrangements of 4 A's at 4 places

$$= \frac{4!}{4!} = 1$$

अब हमारे पास 5 letters बचा है जिसमे 2 L's, 1 H, 1 B and 1 D है।

No. of arrangements of these 5 letters at 5 places

$$= \frac{5!}{2!} = 5 \times 4 \times 3 = 20 \times 3 = 60 \text{ ways}$$

Hence

∴ The number of words in which vowels occupy even places

$$= (1 \times 60) = 60.$$

Q. In how many of the distinct permutations of the letters in 'MISSISSIPPI' do the 4 I's not come together?

Ans: दिया हुआ word MISSISSIPPI में 11 letters है जिसमें 4 I's, 4 S's, 2 P's and 1 M.

∴ Total no. of words formed by the letters of the given word

$$= \frac{11!}{(4!) \times (4!) \times (2!)} = 34650$$

अब there are 4 I's हैं दिया हुआ word में और इसे [IIII] as 1 letter treat करेंगे।

यह single letter with दिया हुआ 7 letters will give us 8 letters out of which there are 4 S's, 2 P's, 1 M and 1 [IIII]

So, the number of all such arrangements in which 4 I's are together

$$= \frac{8!}{(4!) \times (2!)} = 840$$

Thus, the number of words formed when 4 I's are together = 840

Hence, the number of words formed when 4 I's are not together

$$= (34650 - 840) = 33810$$

Q. (i) How many words can be formed with the letters of the word, 'HARYANA'? How many of these

(ii) have H and N together?
(iii) begin with H and end with N?
(iv) have 3 vowels together?

Sol. (i) दिया हुआ word 'HARYANA' में 7 letters हैं जिसमें 1 H, 3 A's, 1 R, 1 Y and 1 N.

Total number of words formed by all the letters of the given word

$$= \frac{7!}{3!} = 840$$

(ii) H, N को एक single letter [HN]

अब, [HN] + ARYAA मिल कर 6 letters होगा जिसमें 3 A's, 1 R, 1 Y & 1 [HN] हैं

(P.T.O.)

इसलिए,

Total number of all such arrangements

$$= \frac{6!}{3!} = 120$$

लेकिन H & N को खुद में हम 2! ways से arrange कर सकते है।

Hence,

the number of words having H & N together

$$= (120 \times 2)$$
$$= 240.$$

(iii) H को पहला position पर और N को last position पर fix करने के बाद मेरे पास 5 letters है जिसमे से 3 A's, 1 R and 1 Y.

(P.T.O.)

Hence,

the no. of words beginning with H and ending with N

$$= \frac{5!}{3!} = 5 \times 4 = 20$$

(iv) have 3 vowels together:

दिया हुआ word में 3 vowels है AAA और इसे [AAA] as 1 letter consider करेंगे।

अब हमें 5 letters को arrange करना है 5 places में i.e. namely HRYN + [AAA].

Hence,

Total No. of words formed having all vowels together

$$= 5! = (5 \times 4 \times 3 \times 2 \times 1)$$
$$= 120$$

Q. In how many ways can the letters of the word 'PERMUTATIONS' be arranged, if

(i) the words start with P and end with S ?

(ii) the vowels are all together?

Ans:

(i) P को extreme left position में fix कर दिया और S को extreme right position में fix कर दिया।

अब, हमारे पास 10 letters बचा जिसमें को T 2 बार आ रहा है, और बाकी सभी अलग है।

इसलिए,

required no. of arrangements

$$= \frac{10!}{2!} = 1814400$$

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(ii) दिया हुआ word में 5 vowels है, namely E, U, A, I, O.

इन पाँचो vowels को EUAIO as one letter, हमें 8 letters namely EUAIO + PRMTTNS को arrange करना है जिसमे T 2 बार आता है और बाकी अलग है।

No. of all such arrangements

$$= \frac{8!}{2!} = 20160$$

अब, 5 vowels सभी अलग है इसलिए वो खुद मे 5! ways से arrange होगा। = 120 ways.

Hence,

the number of arrangements in which 5 vowels are together

$= (20160 \times 120)$

$= 2419200$ ways.

Q. If the different permutations of the word 'EXAMINATION' are listed as in a dictionary, how many items are there in the list before the first word starting with E?

Ans: dictionary में हर stage पर words को alphabetical order में arrange किया जाता है।

जब पहला word 'EXAMINATION' के letters को arrange कर के बनाया गया होगा तो वह होगा AAEIIMNNOTX.

A से शुरू करके हम बाकी 10 letters AEIIMNNOTX जिसमें 2 I's, 2 N's और बाकी सभी different हैं हर possible ways में

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Thus,

the number of words formed beginning with A.

$$= \frac{10!}{(2!)\times(2!)} = 907200.$$

तो dictionary में अगला word होगा
EAAIIMNNOTX.

इसलिए 907200 words है dictionary में जो कि E से start होने वाले word के पहले आते हैं।

Q (i) Find the number of words can be made using all the letters of the word 'AGAIN'.

(ii) If these words are written as in a dictionary, what will be the 50th word. ?

Ans: (i) दिया हुआ word 'AGAIN' में 5 letters है जिसमे 2 A's है और बाकी सभी distinct है।

Hence, the number of words formed by using all the letters of the given word

$$= \frac{5!}{2!} = 60.$$

(ii) जब 'AGAIN' word के letters को dictionary में list करेंगे तो first word होगा AAGIN.

A से शुरुआत करके बाकी बचे 4 letters A, G, I, N, को arrange करने के 4! ways या 4! words i.e. 24 words

~~The~~ 25th word होगा GAAIN जो कि G से start होगा।

अब G से start करके बाकी 4 letters AAIN (जिनमें A 2 बार आया & बाकी सब अलग अलग है।) in all possible ways.

Number of such arrangements

$$= \frac{4!}{2!} = \frac{4 \times 3 \times (2 \times 1)}{2 \times 1} = 12$$

इसलिए, हमने 12 नए words बनाएँ।

तो 37th word होगा IAAGN जो की I से start होगा।

Now, I से start करके बाकी 4 letters को arrange करने के तरीके (AAGN) we get

$$\frac{4!}{2!} = 12 \text{ words.}$$

The 49th word would be NAAGI and hence the 50th word is NAAIG.

Q. How many numbers greater than a million can be formed with the digits 2, 3, 0, 3, 4, 2, 3?

Ans: देखो 1 Million = 10,00,000

इसलिए कोई भी Number जो 1 Million से बड़ा होगा, वह सभी सातों digits contain करेगा।

अब, हमें सात digits arrange करने हैं जिनमें से 2 दो बार आ रहा है और 3 तीन बार आ रहा है और बाकी अलग अलग है।

No. of such arrangements

$$= \frac{7!}{(2!) \times (3!)}$$

$= 420$

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यह सारे arrangements में वो भी संदर्भ शामिल होंगे जिनकी शुरुआती digit 0 होगी।

तो हमने zero को Million's place पर fix कर दिया उसके बाद बचा 6 digits जिनमें 2 दो बार, 3 तीन बार, और बाकी अलग है जो की

$$= \frac{6!}{(2!) \times (3!)} = 60 \text{ ways}$$

में arrange हो सकते हैं।

<u>Hence</u>,

the number of required numbers

$= (420 - 60)$

$= 360$ ways

(P.T.O.)

Q. How many numbers greater than 400000 can be formed by using the digits 0, 2, 2, 4, 4, 5 ?

Ans: हमें six digits दिया जा रहा है और ~~हमें~~ each no. greater than 400000 is a six-digit number, having 4 ~~or~~ 5 in the extreme left position

जब 4, extreme left position में होगा तो, बचा हुआ 5 places को हम 0, 2, 2, 4, 5 digits (जिसमें 2 दो बार और बाकी सभी अलग-अलग)

No. of such numbers $= \frac{5!}{2!} = 60$

जब 5, extreme left position में होगा तो, बचे हुए 5 places को हम 0, 2, 2, 4, 4 (जिसमें 2 दो बार, 4 दो बार और बाकी सभी अलग है).

No. of such numbers $= \frac{5!}{(2!) \times (2!)} = 30$

Hence, the required number of numbers
$= (60+30) = 90.$

Q. How many numbers can be formed using the digits 1, 2, 3, 4, 3, 2, 1 so that the odd digits always occupy the odd places?

Ans: हमे सात digits दिया हुआ है namely 1, 2, 3, 4, 3, 2, 1.

इसलिए, हमे 7-digit no. बनाना है जिस प्रकार odd digits odd places पर आऐ।

हर 7-digit no. के पास 4 odd places है

दिया four odd digits are 1, 3, 3, 1 जिनमे 1 दो बार आता & 3 दो बार आता है।

(P.T.O.)

इसलिए ,

No. of ways to fill up 4 odd places

$$= \frac{4!}{(2!) \times (2!)} = 6.$$

दिया है तीन even digits जो है 2, 4, 2 जिसमे 2 आता है दो बार 4 आता है 1 बार.

इसलिए

No. of ways to fill up 3 even places

$$= \frac{3!}{2!} = 3.$$

Hence, the required no. of numbers

$$= (6 \times 3)$$
$$= 18.$$

Q. Find the total no. of permutations of the letters of each of the words given below:-

(i) APPLE (ii) ARRANGE
(iii.) COMMERCE (iv.) INSTITUTE
(v.) ENGINEERING
(vi.) INTERMEDIATE

Ans:
(i.) 2 p's, 1 A, 1 L, 1 E

$= \frac{5!}{2!} = 5 \times 4 \times 3$
$= 20 \times 3$
$= 60$ ways

(ii.) 2 A's, 2 R's, 1 N, 1 G, 1 E

$= \frac{7!}{2! \times 2!} = \frac{7 \times 6 \times 5 \times \cancel{4} \times 3 \times 2}{\cancel{2 \times 2}}$
$= 42 \times 30$
$= 1260$ ways

(P. T. O.)

(iii.) 2M's, 2E's, 2C's, 1O, 1R

$$= \frac{8!}{2! \times 2! \times 2!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 2}$$

$= 42 \times 20 \times 6$

$= 42 \times 12 \times 10$

$= 5040$ ways

(iv.) 3I's, 2T's, 1N, 1S, ~~1I~~, 1U, 1E

$$= \frac{9!}{3! \times 2!}$$

~~$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{3 \times 2 \times 2}$$~~

~~$= 72 \times 42 \times 30$~~

~~$= 3024 \times 30$~~

~~$= 90720$~~

$= 72 \times 15 \times 28$

$= 1080 \times 28$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(3 \times 2 \times 1) \times (2 \times 1)}$$

$= 420 \times 72$

$= 30240$ ways.

(v.) 3E's, 3N's, 2G's, 2I's, 1R

$$= \frac{11!}{3! \times 3! \times 2!}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(3 \times 2) \times (3 \times 2) \times 2}$$

$=$ ~~19958400~~ 277200 ways.

(vi.) 2I's, 1N, 2T's, 3E's, 1R, 1M, 1D, 1A

$$= \frac{12!}{(2!) \times (2!) \times (3!)}$$

$= 19958400$ ways

Q. In how many ways can the letters of the expression $x^2 y^3 z^4$ be arranged when written without using exponents?

Ans: मेरे पास 2 x's, 3 y's & 4 z's हैं और कुल 9 letters हैं।

∴ Total no. of arrangements

$$= \frac{9!}{(2!) \times (3!) \times 4!}$$

$= 1260$ ways.

Q There are 3 blue balls, 4 red balls and 5 green balls. In how many ways can they be arranged in a row?

Ans: हमें 12 balls को arrange करना है जिसमें से की 3 एक तरह के हैं, 4 दूसरे तरह के हैं & 5 तीसरे तरह के हैं

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इसलिए,

Required no. of arrangements

$$= \frac{12!}{(3!) \times (4!) \times (5!)}$$

Q. A child has three plastic toys bearing the digits 3, 3, 5 respectively. How many 3-digit numbers can he make using them?

Ans: हमें 3 Objects को एक row में arrange करना है जिसमें से 2 are alike of one kind & the third one of its own kind.

∴ Total no. of arrangements

$$= \frac{3!}{2!} = 3$$

(P.T.O.)

Q. How many different signals can be transmitted by arranging 2 red, 3 yellow and 2 green flags on a pole, if all the seven flags are used to transmit a signal ?

Ans: देखो इस question का कहना है कि हम कितने possible ways से signals transmitt कर सकते हैं जिसमें signal सात flag को मिल कर बना हो।

तो तो सात flags हैं उसमें।

2 Red ; 3 yellow & 2 green Flags

Hence,

Required number of signals

$$= \frac{7!}{(2!) \times (3!) \times (2!)}$$

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Q. How many words can be formed by arranging the letters of the word 'ARRANGEMENT', so that the vowels remain together?

Ans: मान लिया vowels [AAEE] को की यह एक single letter है।

अब, [AAEE] + RRNGMNT हैं में 8 letters होगा। जिसमें 2 R's, 2 N's, 1 G, 1 M & 1 T हैं।

No. of these arrangements

$$= \frac{8!}{(2!) \times (2!)} = 10080.$$

अब, AAEE के पास 4 letters हैं जिसमें से कि 2 A's और 2 E's हैं।

No. of their arrangements

$$= \frac{4!}{(2!) \times (2!)} = 6.$$

Required no. of words formed

$= (10080 \times 6)$

$= 60480$ words.

Q. How many words can be formed by arranging the letters of the word 'INDIA', so that the vowels are never together?

Ans: Word 'INDIA' में 2I's, 1A, 1N and 1D.

No. of permutations of the letters of the given word

$$= \frac{5!}{2!} = 60$$

मान लिया कि vowels [IIA] को एक letter माना.

अब [IIA] + ND हैं 3 letters, जो सभी अलग हैं।

No. of their arrangements = 3! = 3×2×1 = 6

Now, IIA has 3 letters, out of which there are 2 I's and 1 A.

Number of their arrangements = $\frac{3!}{2!}$ = 3.

No. of words having all vowels together = (6 × 3) = 18.

Hence,

No. of words in which all vowels are not together

= (No. of words that can be made from the given word) − (No. of words having all vowels together)

= 60 − 18

= 42.

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Q. Find the no. of arrangements of the letters of the word 'ALGEBRA' without altering the relative positions of the vowels

Sol: दिए हुए word में 3 vowels है namely A, E, A जिसमें की 2 A's और 1 E हैं।

No. of arrangements of the vowels

$= \frac{3!}{2!} = 3.$

दिए गए word में 4 consonant है namely L, G, B, R.

No. of arrangements of the consonants

$= 4! = 24.$

∴ Required no. of arrangement

$= (3 \times 24)$

$= 72.$

Q. How many words can be formed from the letters of the word 'SERIES', which start with S and end with S?

Ans: S को शुरुआत मे fix करने और अंत मे fix करने के बाद हमारे पास बचा 4 letters, जिसे हमे arrange करना है जिनमे ~~2A's & 1E~~ 2 E's, 1R & 1I.

No. of their arrangements

$$= \frac{4!}{2!} = 12$$

Q. In how many ways can the letters of the word 'PARALLEL' be arranged so that all L's do not come together?

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Ans: दिया हुआ word 'PARALLEL' में 8 letters है जिसमें 2 A's, 3 L's, 1 P, 1 R and 1 E।

No. of their arrangements

$$= \frac{8!}{(2!) \times (3!)} = 3360$$

मान लिया कि [L, L, L] एक letter है,

तब [L,L,L] + PARAE के पास 6 letters है जिसमें 2 A's और बाकी सभी distinct हैं।

No. of their arrangements

$$= \frac{6!}{2!} = 360.$$

Now,

Number of arrangements in which 3 L's are not together
= (3360 − 360) = 3000 ways.

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Q. How many different words can be formed with the letters of the word 'CAPTAIN'? In how many of these C and T are never together?

Ans: दिया गया word 'CAPTAIN' में 7 letters हैं जिसमें 2 A's और बाकी सभी different हैं

No. of their arrangements $= \frac{7!}{2!}$
$= 2520$

मान लिया ☐☐☐ को एक letter मान

अब ☐☐☐ + 1

मान लिया की [CT] एक letter है
तब, [CT] + APAIN के पास 6 letters जिसमें से 2 A's और बाकी सभी distinct.

No. of their arrangements $= \frac{6!}{2!} = 360$

Also, CT can be arranged among themselves in 2!
= 2 ways.

Number of arrangements in which C and T are together

= (360 × 2) = 720

Then,

Number of arrangements in which C and T are never together
= (2520 − 720)
= 1800

Q. In how many ways can the letters of the word 'ASSASSINATION' be arranged so that all S's are together.

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Ans: दिया गया word 'ASSASSINATION' मे 3A's, 2I's, 2N's, 4S's, 1T & 1O.

हम [SSSS] को एक letter consider करेंगे.

तब [SSSS] + AAAIINNTO के पास 10 letters है जिसमे से 3A's, 2I's, 2N's, 1T, 1O & 1 [SSSS].

No. of their arrangements

$$= \frac{10!}{(3!) \times (2!) \times (2!)} = 151200$$

Q. (i) How many arrangements can be made by using all the letters of the word 'MATHEMATICS'?
(ii) How many of them begin with C?
(iii) How many of them begin with T?

Ans: (i) MATHEMATICS

11 words

No. of arrangements that can be made by using the letters of 'MATHEMATICS'

$$= \frac{11!}{2!\cdot 2!\cdot 2!} = 4989600$$

(ii) मान लिया की हर एक arrangement C से शुरु होता है, तब बचे हुए 10 letters में 2 M's, 2 A's, 2 T's और बाकी सभी distinct हैं।

No. of all such arrangements

$$= \frac{10!}{2! \times 2! \times 2!}$$

$$= 453600$$

(iii.)

मान लिया है कि arrangement T से शुरू होता है। तब बचे हुए 10 letters में 2M's, 2A's और बाकी सभी distinct हैं।

Number of all such arrangements

$$= \frac{10!}{(2!) \times (2!)} = 907200$$

Q. In how many ways can the letters of the word 'INTETERMEDIATE' be arranged so that:-

(i) the vowels always occupy even places?

(ii) the relative orders of vowels and consonants do not change?

(P.T.O.)

Ans: (i) दि गई word 'INTERMEDIATE' में 12 letters हैं namely 6 vowels and 6 consonants

क्योंकि vowels को even places को occupy करना है इसलिए

N □ T □ R □ M □ D □ T □

type of arrangements, जहाँ vowels, even places को occupy करेंगे।

इसमें 6 consonants हैं namely 2 T's, 1 N, 1 R, 1 M and 1 D.

No. of arrangements of 6 consonants

$$= \frac{6!}{2!} = 360$$

The given word has 6 vowels, namely 2 I's, 3 E's and 1 A

No. of arrangements of 6 vowels

$$= \frac{6!}{(2!) \times (3!)} = 60.$$

∴ total no. of words formed

$$= 360 \times 60 = 21600$$

(ii.) the relative orders of vowels and consonants do not change?

दिया हुआ है, की हम vowel को दूसरे vowel से replace कर सकते हैं और हम consonant को दूसरे consonant से replace कर सकते हैं. given herewith.

No. of arrangements of consonants

$$= \frac{6!}{2!} = 360$$

No. of arrangements of vowels

$$= \frac{6!}{3! \times 2!} = 60$$

∴ Total number of words formed

$$= (360 \times 60)$$
$$= 21600.$$

Q. (i) Find the number of different words formed by using all the letters of the word, 'INSTITUTION'.

In how many of them,

(ii) are the three T's together?

(iii) are the first two letters the two N's?

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Ans: (i) 'INSTITUTION' word में 11 letters हैं जिसमें 3 I's, 3 T's, 2 N's, 1 S, 1 U & 1 O.

Total number of words

$$= \frac{11!}{(3!) \times (3!) \times (2!)} = 554400$$

(ii) [TTT] को एक letter मान लिया तो, दिया हुआ word को हम ऐसे express करेंगे as [TTT] + INSIUION, जिसमें 9 letters हैं जिनमें 3 I's, 2 N's, 1 S, 1 U & 1 O.

No. of words in this case

$$= \frac{9!}{(3!) \times (2!)}$$

$$= 30240$$

(iii) are the first two letters the two N's

NN को as first letter रखे, उसके बाद बचे 9 letters को arrange करना है। जिसमे 3 I's, 3 T's, 1 S, 1 U and 1 O.

∴ Number of words formed

$$= \frac{9!}{(3!) \times (3!)} = 10080$$

Q. How many five-digit numbers can be formed with the digits 5, 4, 3, 5, 3?

Ans: □ □ □ □ □

हमें 5-digit numbers बनाना है जिसमें '5' 2 बार आता है, '3' 2 बार आता है और '4' 1 बार आता है।

∴ Required

$$= \frac{\quad}{(2!)}$$

Q. How many ... formed ... 2, 3, 4, 5, 4, ... the odd dig... odd places

Ans: हमारे पास 3, 5, 3 जिन... एक बार।

Number of ... odd place...

=

हमारे पास ... namely 2, ... 2 occurs ...

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∴ Required number of numbers

$$= \frac{5!}{(2!) \times (2!) \times (1!)} = 30.$$

Q. How many numbers can be formed with the digits 2, 3, 4, 5, 4, 3, 2 so that the odd digits occupy the odd places?

Ans: हमारे पास 3 odd digits, namely 3, 5, 3 जिनमें से 3 दो बार और 5 एक बार।

Number of ways to fill the odd places

$$= \frac{3!}{2!} = 3.$$

हमारे पास चार even digits हैं namely 2, 4, 4, 2 out of which 2 occurs twice & 4 occurs twice.

Number of ways to fill the even places.

$$= \frac{4!}{(2!) \times (2!)} = 6.$$

Hence,

Required number of numbers

$$= (3 \times 6) = 18.$$

Q How many 7-digit numbers can be formed by using the digits 1, 2, 0, 2, 4, 2, 4 ?

Ans: □ □ □ □ □ □ □

No. of word that can be formed using the given digits

$$= \frac{7!}{3! \times 2!} = 420$$

इसमें wo भी No. included जो 0 से शुरुआत होते हैं; इसलिए उनको हम निकाल कर total no. of arrangements से subtract कर सकेंगे :-

No. of words that can be formed using given digits that starts with zero:

$$= \frac{6!}{3! \times 2!} = 60.$$

Hence

Total 7-digit numbers (valid)

$$= \left(\frac{7!}{3! \times 2!} - \frac{6!}{3! \times 2!} \right)$$

$$= 420 - 60$$

$$= 360 \text{ numbers}$$

Q. How many 6-digit numbers can be formed by using the digits 4, 5, 0, 3, 4, 5 ?

Ans: □ □ □ □ □ □

Total no. of digits that can be formed using given digits

$$= \frac{6!}{(2!) \times (2!)}$$

&

No. of digits which start with zero

$$= \frac{5!}{(2!) \times (2!)}$$

Hence,

Total no. of valid 6-digit numbers

$$= \frac{6!}{2! \times 2!} - \frac{5!}{2! \times 2!} = 180 - 30 = 150$$

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Circular Permutations

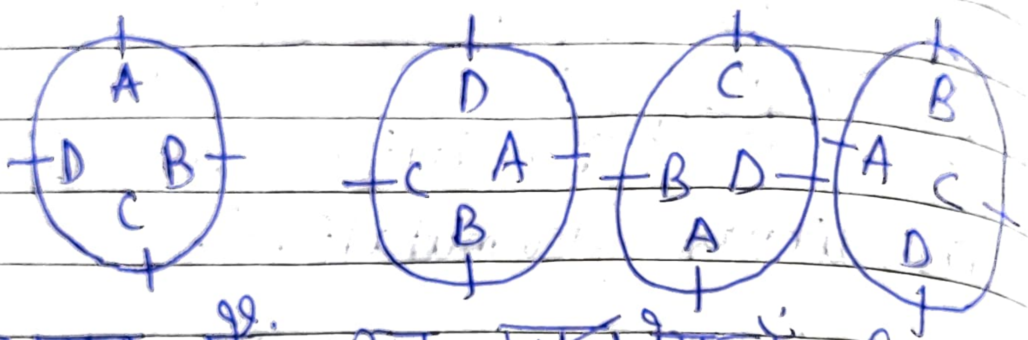
अभी तक हमने सिर्फ arrangements उन objects का किया था जो एक line में arrange किए जा रहे थे। इस प्रकार के permutations are known as "linear permutations".

अब, बजाय objects को line में arrange करने के, अगर मैं उन्हें एक circle के form में arrange कर देता हूँ तो उसे "circular permutations" कहते हैं।

Circular permutations में जो महत्व रखता है वह यह की एक object का position other objects के respect में।

अगर मैं एक linear permutation consider करूँ
ABCD, BCDA, CDAB & DABC
तो यह साफ बात है की ये सभी अलग हैं।

अब मैं A, B, C, D को एक circle के circumference के along arrange कर देता हूँ।



अगर मैं यह मान लूं कि position एक objects का दूसरे के respect में तो हमें यह पता चला की सभी arrangements same हैं।

example

अगर चार आदमी A, B, C, D एक round table के चारों ओर बैठते हैं तो; जैसा की first arrangement में दिखाया है, तब अगर हरेक एक कुर्सी आगे shift हो जाएँ तो हमें दूसरा arrangement मिल जाएगा

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इस arrangement में हरएक person के पास वही पुराने Ist arrangement वाले परोसी हैं, left की ओर और right की ओर।

इसलिए, पहला और दूसरा दोनों same circular permutation हैं। Same case है third & fourth arrangements के साथ जो ऊपर दिए हुए हैं।

इसलिए, circular permutation में हम पहले एक object का position fix करेंगे और उसके बाद बाकी objects को arrange करेंगे।

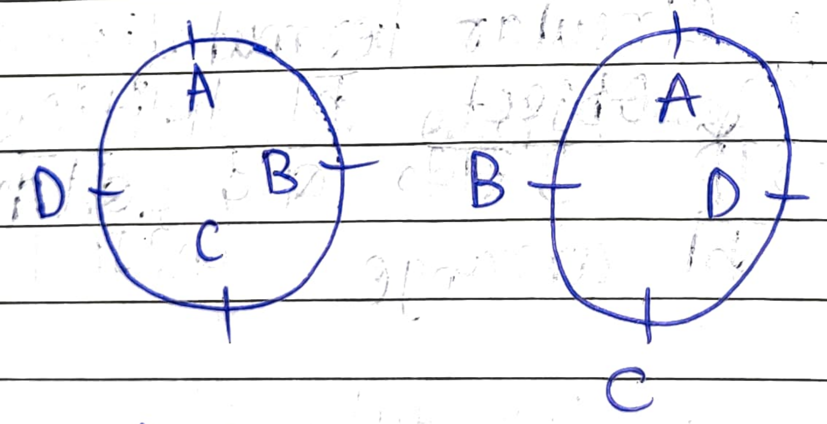
There are two types of circular permutations

(i) ऐसा circular permutation जिसमें clockwise और anticlockwise arrangements अलग-अलग permutations माने जाएँ।
e.g.: seating arrangement of person round a table.

(ii) ऐसा circular permutation जिसमें clockwise & anticlockwise same permutation होते हैं।

e.g: arranging some beads to form a necklace.

Look at the circular permutations, given below:-



मान लो की A, B, C, D चार beads हैं जो एक necklace बना रही है।

उन्हें clockwise और anticlockwise direction में arrange किया है पहले & दूसरे arrangements में।

अगर necklace जो की first arrangement था अगर हम उसको घुमा दे clockwise to anticlockwise तो हमें दूसरा arrangement मिल जाएगा।

इसलिए, There is no difference between the above two arrangements.

Results on circular permutations

Theorem 1: The number of circular permutations of n different objects is $(n-1)!$

Proof: एक object को एक position में fix कर सकते है n ways में।

Thus, each circular permutation corresponds to n linear permutations, depending upon where from we start.

$\because$ क्योंकि मेरे पास $n!$ linear permutations है यह बोलता है की यहां $\frac{n!}{n}$, i.e., $(n-1)!$ circular permutations है।

Theorem 2: The number of ways in which 'n' persons can be seated round a table is $(n-1)!$

Proof: एक person का position fix कर लिया और फिर बचे $(n-1)$ persons को सभी possible ways में arrange किया तो $(n-1)!$ में होगा।

Theorem 3: Show that the number of ways in which n different beads can be arranged to form a necklace is

$$\frac{1}{2}(n-1)!$$

Proof: एक bead का position fix कर दिया उसके बाद बचा हुआ $(n-1)$ beads को $(n-1)!$ ways में arrange करेंगे

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अगर arrange करने मे clockwise & anticlockwise मे कोई अंतर नहीं है

∴ the required no. of ways

$= \frac{1}{2}(n-1)!$

Q. In how many ways can 8 students be arranged in a circle?

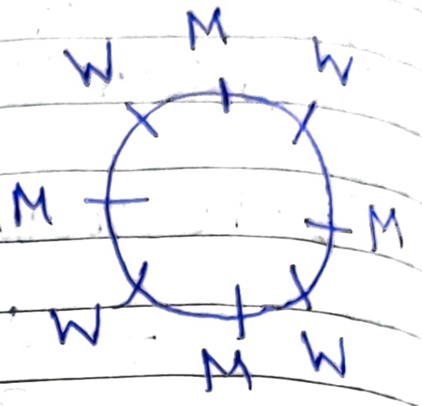
Ans: ∴ No. of ways in which 8 students can be arranged in a circle

$= (8-1)!$
$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 5040.$

Q. In how many ways can a party of 4 men and 4 women be seated at a circular table so that no two women are adjacent?

Ans: 4 Men एक circular table पर ऐसे बैठेंगे कि दो Men के बीच एक vacant seat हो।

No. of ways in which 4 men एक circular table पर बैठे



$= 3! = 3 \times 2 = 6$

अब, बचे हुए 4 सीट 4 women के द्वारा occupied होगा

$^4P_4 = 4! = 24$ ways.

$\therefore$ The required no. of ways $= 6 \times 24$

$= 144$ ways.

Q. If 20 persons were invited for a party, in how many ways can they and the host be seated at a circular table?

Ans. सीधी बात है की 21 persons को एक round circular table पर बैठाना है

हम पहले एक person को बिठा कर starting point fix कर देते हैं। (मान लो की host को बिठा दिया)

बचा हुआ 20 persons को 20! ways से arrange कर सकेंगे। ~~20! ways~~

Hence,

The number of ways in which these 21 persons can be seated round a circular table

$= 20!$

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Q. If 20 persons were invited for a party, in how many ways can they and the host be seated at a circular table such that two particular persons be seated on either side of the host?

Ans:

No. of persons to be seated on the round table = 21.

∴ No. of total ways in which these 21 persons can be seated round a circular table

$$= (21-1)! = 20!$$

अब दो persons, host के दोनों में दो तरीकों से बैठ सकते हैं और बचे 18 persons circular table पे 18! ways से बैठ सकते हैं।

∴ No. of ways of seating 21 persons at a circular table with two particular persons on either side of the host

a
ey

$= (2 \times 18!)$

Q. A round table conference is to be held between delegates of 20 countries. In how many ways can they be seated if two particular delegates may wish to sit together?

Ans. मान लिया की ये दो delegates एक है अब बचे 19 delegates को circular table पर 18! ways से arrange करेंगे।

यह दो delegates को आपस मे 2! ways से arrange कर सकेंगे।
$= 2 \times 1$ ways.

∴ the required number of ways
$= 2 \times (18!)$ ways.

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Q. In how many ways can 7 persons sit around a table so that all shall not have the same neighbours in any two arrangements?

Ans: 7 persons को एक round table पर 6! ways.

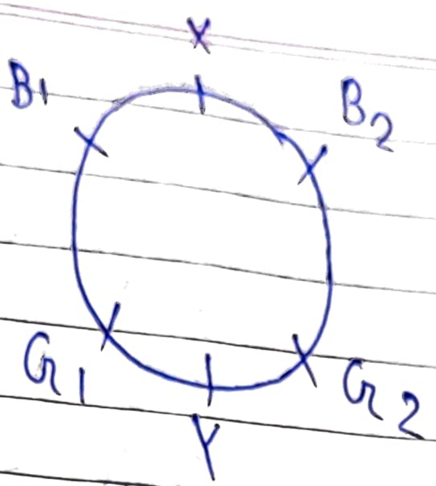
लेकिन, clockwise & anticlockwise arrangements, हर एक person के पास same neighbour है।

∴ Required no. of ways

$$= \frac{1}{2} \times 6! = 360$$

Q. 3 boys and 3 girls are to be seated around a table in a circle. Among them, the boy X does not want any girl neighbour and the girl Y does not want any boy neighbour. How many such arrangements are possible?

Ans: X और Y position को fix कर दिया



अब,

boys B_1, B_2 & girls G_1, G_2 may have their neighbours X and Y respectively.

boys B_1 & B_2 खुद में 2! ways से arrange किए जा सकते हैं & girls G_1 & G_2 खुद में 2! ways से arrang किए जा सकते हैं।

Hence, the required no. of arrangements
$= (2 \times 2)$
$= 4.$

Q. In how many ways can 6 persons be arranged in a circle?

Ans: The no. of ways in which 6 persons can be arranged in a circle
$= (6-1)!$
$= 5! = 120$ ways.

Q. There are 5 ~~men~~ men and 5 ladies to dine at a round table. In how many ways can they seat themselves so that no two ladies are together?

Ans: देखो सबसे पहले 5 girls को एक circular table पर alternate यानी दो गर्ल्स के बीच में एक seat खाली छोड़ते हुए बैठा दिए ; अब 5 girls को हम एक round table पर (5−1)! i.e. 4! ways में arrange कर सकते हैं।

अब बचा 5 places, इन बचे 5 place में 5 boys को बैठाने का तरीका 5! ways.

इसलिए,

∴ No. of ways in which they seat themselves so that no two ladies are together

$= 5! \times 4!$

$= 24 \times 120 = 2880$ ways

(P_1, P_2)

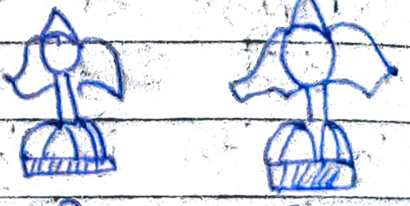
अब initals में word same नहीं होता इसलिए,
P_1 place को भरने के लिए हमारे पास 5 ways & P_2 place को भरने के लिए हमारे पास 4 ways

$$= (5 \times 4)$$
$$= 20 \text{ ways.}$$

Q. Ten students are participating in a race. In how many ways can the first three prizes be won?

Ans: Total no. of participants = 10

(3 prizes)



P_1 P_2

No. of ways can the first three prizes be won